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PLANNING, SCHEDULING, AND EXPEDITING ENGINEERING PROJECTS WITH THE AID OF ELECTRONIC COMPUTERS

by

John Corbin Pollock

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ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
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Particle Accelerator Division

Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Mechanical Engineering
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Illinois Institute of Technology

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PREFACE

This thesis has grown out of investigations begun at the Argonne National Laboratory, Argonne, Illinois, in June, 1961, to develop a method of using electronic computers to evaluate logical network diagrams representing the engineering and construction effort necessary to build the Zero Gradient Synchrotron. To date, the computer outputs furnish information which aids in planning the sequence of activities, determining the intervals of time in which the activities must be performed, and in measuring the criticality of each activity to the timely completion of the project. Provisions for predicting manpower requirements by craft, as a function of time, have also been incorporated into the analysis.

Investigative work in this field has been under way for about five years by several organizations and much progress has been made. However, the method is still not completely developed nor is the range of its applicability entirely known.

It is my purpose to draw together existing material on the subject, comment on its validity in the light of personal experience, and present the specific development and implementation of network analysis techniques being used in the construction of the Zero Gradient Synchrotron.

I am indebted to Mr. Willard Hanson, of the Argonne National Laboratory, who originally conceived this investigation and who has furnished support and encouragement during its pursuit, and to Professor Roland Budenholzer of the Illinois Institute of Technology for his advice and guidance.

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CHAPTER I
INTRODUCTION

Today, much engineering work is defined in terms of projects. A project is a partially ordered set of activities directed towards the accomplishment of a specific objective. The partial ordering arises from technological and environmental restrictions that force certain jobs to be done before others can be started.

During the latter quarter of the nineteenth and the first half of the twentieth century, management has acquired considerable experience in the control of "continuing businesses" characterized by continuous production. These planning and control methods have assumed the existence of historical data since they involve the measurement of past processes. The worker's task is defined by: (1) prescribing the exact order and method of work; (2) prescribing the tools and equipment to be used; and (3) establishing the time in which the task is to be accomplished. The first two pieces of information are obtained through motion study, the last through time study. One-time research and development or construction projects do not lend themselves to this type of analysis.

The most common method of project scheduling in use today is the bar chart developed by Henry Lawrence Gantt. A bar (implying the existence of an activity) of a length proportional to the expected duration of the activity, is placed on a horizontal time scale at a time corresponding to the scheduled date of starting. These charts are limited, since it is not immediately apparent which activities constrain other activities. In practice, because of the complex nature of bar-chart representation, it is necessary to simplify the presentation. The tendency is to lump several activities together into one gross activity. In most cases this leads to oversimplification in which activities which significantly constrain the project may be overlooked

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Today, projects are growing in scope and complexity, and there is increased pressure to compress the time required to obtain the project's goal. The project manager must be able to coordinate the various departments within his organization and to supply rapid and accurate information on progress, deliveries, resources, and costs. The sheer complexity of the operation forces him to separate himself deliberately from matters of detail and to deal with the broader aspects of the problem. The several groups concerned with the work do their own planning and scheduling, largely independently of one another. Project schedules are often derived from gross estimates of total requirements. It is obvious that any effective method of planning must ensure that the tremendous complexity of coordinating many diverse activities does not force an artificial over-simplification and result in errors of omission, errors of logic, and errors of method.

During the past five years, two distinct, but in many ways similar, methods for planning, scheduling, and monitoring engineering and scientific research and development projects have been evolved. They are the Critical Path Method and PERT.

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CHAPTER II
HISTORICAL DEVELOPMENT

Critical Path Method

The earliest work in the field of using logical network diagrams to aid in planning and scheduling projects appears to have been done by the E. I. du Pont de Nemours and Company. In 1956, the Integrated Engineering Control Group of du Pont (Wilmington, Delaware) began to explore possible alternatives to traditional methods of project scheduling. They hoped to develop a method which would integrate all activities in a project towards a common goal, pinpoint potential difficulties well in advance, and permit management by exception. It was realized early in the effort that the generation of such a plan would require the consideration of information in greater detail than had been previously attempted. In 1957, the group initiated a survey on the possibilities of using electronic computers to process the data needed in such an analysis. Remington Rand's UNIVAC division was requested to assist in this analysis as part of their customer service. A team of engineers, headed by Morgan R. Walker of du Pont, and a team of mathematicians and computer experts from Remington Rand, headed by James E. Kelley, worked through 1957 and produced what is now known as the Critical Path Method (CPM). The method derives its name from the significance that is attached to the chain of critical activities that determines the project's duration.

The central idea of the Critical Path Method was the topological representation of a project in the form of a logical network diagram. Each activity or job necessary to the completion of the project was represented by an arrow. The arrows, interconnected at nodes, showed the technological and planned relationships between activities.

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Pilot tests of CPM were run in 1957. The results showed that a full-scale application was warranted. The project chosen was the construction of a chemical plant valued at ten million dollars. In order to obtain an adequate basis for evaluating the effectiveness of CPM, it was decided that the group using it would operate independently of the normal scheduling group. The CPM team's schedules were not to be used in actually administering the project. The project was broken down into five hundred and forty-nine activities, ranging in cost from fifty to fifty thousand dollars. There were, in addition, two hundred and ninety-seven dummy jobs,[1] for a total of eight hundred and forty-six.

The original CPM program had been written for UNIVAC I. The CPM teams went into more detail in their analysis than had been originally anticipated, thereby making it necessary to reprogram the problem for a larger computer, the Remington Rand 1103A. In March 1958, a management decision at du Pont caused a forty percent change in the project plan. Updating the network diagrams took about ten percent of the original effort expended in preparing them. The conventional scheduling group expended almost one hundred percent of the original effort. With only thirty percent of the design information, the CPM group accurately predicted the total manpower curve.

The normal group, early in the project, determined what they thought would be the critical deliveries. The CPM group included all one hundred and fifty-six deliveries. Their analysis showed that only seven of these were critical and that three of these were not included by the normal group.

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By July of 1958, a second test of CPM, on a project capitalized at two million dollars, had been completed. At this time it was felt necessary to reprogram the problem for a still larger computer, the Remington Rand 1105.

The third test was at du Pont's Louisville, Kentucky Works, which produces a self-detonating material used in the manufacture of neoprene. Because of the hazardous nature of the substance produced, the plant had to be shut down during periodic maintenance and overhaul. It was desired to minimize the down-time as much as possible. It was anticipated that preparing network diagrams for a project of this type would be very difficult, since it was not known exactly what maintenance and replacement operations were necessary until after the equipment had been disassembled and examined. This problem was partially solved by preparing several network diagrams, one for each of several shut-down situations. Unexpected activities, in most cases, were absorbed in the slack associated with the noncritical activities.

By March 1959 this test was finished. Average shut-down time was reduced from one hundred and twenty-five hours to ninety-three hours. It was estimated by du Pont that this savings alone would save the company more than five times the original cost of developing the Critical Path Method.

PERT (Program Evaluation Review Technique)

PERT is a project-monitoring method developed by the United States Navy to aid in the management of the Fleet Ballistic Missile (FBM) weapons system development. The work was begun in January 1958, by the Program Evaluation Branch of the Special Projects Office, Bureau of Ordnance (now Bureau of Weapons), U. S. Navy.

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At this time a schedule for developing the Polaris Missile and associated support equipment had been prepared. It contained thousands of activities and extended years into the future. The need to develop the weapons system was urgent and, as a result, some highly uncertain research and development work had been compressed into short time intervals. Slippages of some scheduled dates had already occurred, and it appeared that the ability of the Program Evaluation Branch to predict future slippages was not sufficient. It appeared that a new management approach was needed. The research team designated their problem as PERT, an acronym for Program Evaluation Research Task. The title was later changed to the present one, Program Evaluation Review Technique. The team consisted of members from the Special Projects Office, the Lockheed Missile Systems Division, and the Booz, Allen and Hamilton Company.

The PERT method was to provide: (1) a measure of current status against approved plans and schedules; (2) a forecast of future progress and problem areas, with probabilities of meeting schedules for planned effort; and (3) a method for evaluating the effects of proposed changes in plans on established schedules for meeting program goals. It was also desired that the new method be compatible with existing reporting procedures then in use: Milestone Reporting and Line of Balance.

Milestones are significant events in the history of a project whose scheduled attainment is used as an index of project progress. Line of Balance, sometimes called Production Analysis, is a system used to analyze the final production of end items. It can point out shortcomings in production plans, but is not suitable for one-time research and development projects.

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The PERT team felt that to describe a project accurately it was necessary to:

1. Select specific identifiable events that are planned to occur along the way to the successful conclusion of the project.
2. Link the planned events so as to graphically portray the interdependencies among them.
3. Estimate the times necessary to move from event to event together with a measure of the uncertainties involved.

This led, as in the case of the Critical Path Method, to the use of network diagrams.

In order to obtain a measure of the uncertainty in time associated with reaching each event, it was assumed that the probability distribution of the time required to pass from one event to another was approximated by a beta distribution. It was also assumed that only three time estimates (optimistic, most likely, and pessimistic) were necessary to determine the expected value and variance of the distribution. The expected value became the time expected to elapse between two given events and the variance an indication of the uncertainty of the estimate. Preliminary analysis indicated that the entire FBM analysis would include from five thousand to ten thousand events, indicating the need for high-speed computing equipment. A survey of available computer installations narrowed the field down to two: the UNIVAC 1103-A at the Applied Physics Laboratory, Johns Hopkins University, and the IBM Naval Ordnance Research Computer (NORC) at the Naval Weapons Laboratory, Dahlgren, Virginia. Because of lower cost, and greater speed and flexibility, the NORC computer was finally chosen. The original NORC consisted of a two-thousand-word, cathode-ray tube, high-speed memory, eight high-speed magnetic tape memory units, and two line-at-a-time printers plus associated card-to-tape-to-card converters and card

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The computer programming was done by programmers at the NORC installation supervised by a staff member from Booz, Allen and Hamilton. Approximately two hundred and twenty-one man-days of effort and $20,645 were required to prepare the ten different programs which constituted a complete processing run.

While the programming effort was going on, data were gathered on the missile subsystem and the propulsion component. These data were used as the basis for a preliminary analysis. The purposes of this test were to determine the availability of the necessary information and to gain insight into the magnitude of the quantities involved. Events were selected from Special Projects Program Management Plans, the Lockheed Master Development Plan, and the Lockheed Master Test Plan.

The events in the missile subsystem were of a high level. The propulsion component was analyzed in greater detail, the network containing one hundred and sixty events.

A second examination of the propulsion component was made while at the same time extending the analysis to the flight control, ballistic shell, re-entry body and guidance components.

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CHAPTER III
NETWORK DIAGRAMS

In analyzing any project, the operations of planning and scheduling must be separated. Simple as this sounds, it is often not done. Planning is defined as determining the relationship between activities and the sequence in which they are to be performed. Scheduling involves assigning expected durations to each activity and determining when they are to start and finish.

The first step, then, is to create a master plan which should embody the following characteristics:

1. Follows a uniform system which is understood by all.
2. Provides a disciplined basis for planning the project.
3. Shows the logical inter-relationship between activities and the influence of external constraining activities.
4. Provides a quick and effective means to evaluate progress to date in which activities and events are defined in a positive sense so that there is no question as to their successful completion.
5. Is an effective means to familiarize new personnel with the scope and detail of the project.
6. For large projects is translatable to a form suitable for data processing.

It is characteristic of any project that the work must be performed in a well-defined order. The relationship of one activity to another can be shown graphically. This graphical representation is called a network diagram. On the diagram, each activity is represented by an arrow (see Fig. 1). The arrow depicts:

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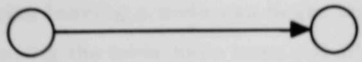


Fig. 1. Activity Representation

Every arrow begins and ends at a node. The node may be thought of as an event in time, such as the beginning or ending of an activity.

The concept of an activity or event must be unambiguously defined. An activity is any time-consuming operation, function, or procedure which begins and ends at identifiable points in time and is essential to the completion of the project. An event is a point in time, the attainment of which requires the completion of one or more activities.

Listed below are examples of typical activities:

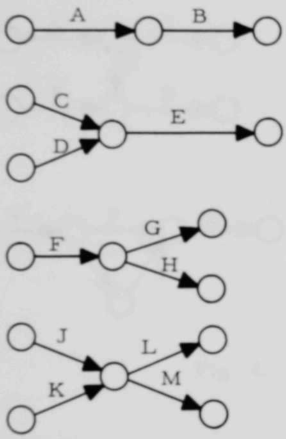
1. planning operations, such as engineering studies and design;
2. time required for the allocation of funds;
3. procurement and training of personnel;
4. procurement operations, such as specification preparation, bidding periods, contract preparation, fabrication, and shipment;
5. fabrication operations, such as engineering studies and design;
6. lead times, e.g., time required to pass until the beginning of suitable weather or the scheduled beginning of the project.

After the essential activities have been determined, the arrows representing them are drawn and interconnected to show the sequence in which the activities are to be performed. The following rules govern the relationship between arrows and nodes.

1. No activity leaving a node can begin until all of the activities entering the node have been completed.
2. All activities entering a node must have the same immediate successors.
3. All activities leaving a node must have the same immediate predecessors.

The rules can be represented symbolically as in Fig. 2.

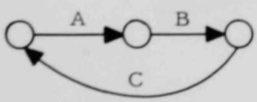
SYMBOL — INTERPRETATION



1. B cannot start until A is completed.
2. E cannot start until C and D are completed.
3. F must be completed before G or H can start.
4. Both J and K must be completed before either L or M can start.

Fig. 2. Diagram Interpretation

It is obvious that no activity can be its own predecessor. An activity such as in Fig. 3 is forbidden.



C is its own predecessor

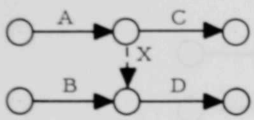
Fig. 3. Loop Error

To make the information on the diagram suitable for data processing, to insure a unique representation for each job, and to aid in efficient reporting, two additional restrictions must be added to the arrow-node relationships:

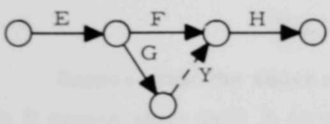
4. No two activities can begin and end at the same node.
5. No activity can be represented on the same diagram more than once.

To preserve the logical relationship between jobs and to satisfy the previous two restrictions, dashed arrows called dummy activities (see Fig. 4) are used. Dummy activities by definition have a duration of zero.

SYMBOL — INTERPRETATION

1. 

Both A and B must be finished before D can start. Activity A must be finished before C can start. Activity C does not depend on B. Activity X is the dummy activity.

2. 

Activity E must be finished before F and G can start. Activity H cannot start until F and G are finished. Activity Y is the dummy activity.

Fig. 4. Dummy Activities

To illustrate the use of dummy activities, consider two examples.

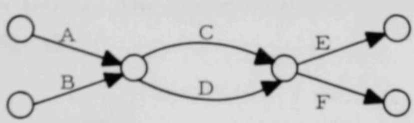


Fig. 5. Example I

The diagram in Fig. 5 violates the rule which states that no two jobs can begin and end at the same nodes, as is indicated for C and D. A correct diagram would be that in Fig. 6.

4. No two activities can begin and end at the same node.
5. No activity can be represented on the same diagram more than once.

To preserve the logical relationship between jobs and to satisfy the previous two restrictions, dashed arrows called dummy activities (see Fig. 4) are used. Dummy activities by definition, have a duration of zero.

SYMBOL	INTERPRETATION
1.	Both A and B must be finished before D can start. Activity A must be finished before C can start. Activity C does not depend on B. Activity X is the dummy activity.
2.	Activity E must be finished before F and G can start. Activity H cannot start until F and G are finished. Activity Y is the dummy activity.

Fig. 4. Dummy Activities

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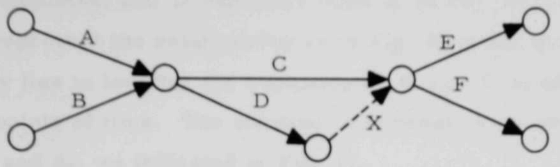

Fig. 6. Solution of Example I

Notice that the logical relationship is preserved. Jobs E and F still cannot start until jobs C and D have been completed.

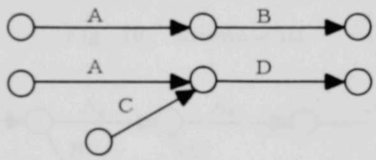

Fig. 7. Example II

Suppose that the above relationship depicted in Fig. 7 exists. Job B cannot start until A is completed. Job D cannot start until Jobs A and C are completed. The diagram as it stands violates the rule which says that no job can be represented more than once. Job A is represented twice. A possible solution might be as indicated in Fig. 8. This solution is incorrect, however, since it was postulated that Job B does not depend on Job C. The correct solution is to introduce a dummy job (see Fig. 9).

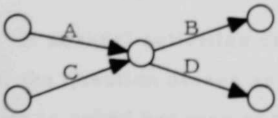

Fig. 8. Incorrect Solution of Example II

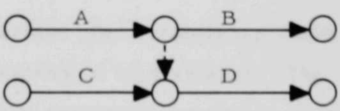

Fig. 9. Correct Solution of Example II

Suppose there exist four activities: A, B, C, and D. Activity B can start when A is 20 percent completed; C can start when A is

60 percent completed; and D can start when A is 100 percent completed. To represent the relationship as in Fig. 10 is not quite correct. The difficulty lies in locating the beginning of B and C at clearly identifiable points of time. The solution is to break A up into three jobs A_1, A_2, and A_3, as indicated in Fig. 11.

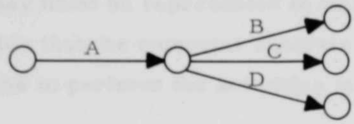

Fig. 10. Example III

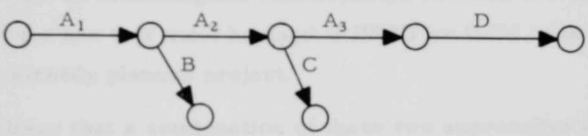

Fig. 11. Solution of Example III

Network diagrams for large projects appear at first glance to be quite complicated; however, they can be constructed in a simple fashion. The diagram is built up by sections, one arrow at a time, by asking and answering the following questions for each job:

1. What immediately precedes this job?
2. What immediately follows this job?
3. What jobs are to be performed concurrently with this job?

If several activities can be performed concurrently but need not be, the question arises as to how they should be displayed. The question is asked because of the amount of resources needed to perform the activities at the same time. Given three activities A, B, and C with no technological ordering, it is possible to perform them in six different sequences: ABC, ACB, BCA, BAC, CAB, and CBA. However, because of manpower limitations, it may not be possible to

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The second approach is to take resource limitations into consideration initially and to construct the network diagram showing planned as well as technological relationships between activities. This approach is the one that must be used if PERT or CPM is being used to monitor an already planned project.

I believe that a combination of these two approaches is the most desirable. The Zero Gradient Synchrotron considered as a project consists of many sub-projects, each the responsibility of a single engineer or scientist. When determining whether or not activities can occur concurrently four questions are asked:

1. Is it technologically feasible for the activities to take place at the same time?
2. If the activities will take place within the same physical area, is there enough space available for them to be performed at the same time?
3. Does the performance of one activity create a hazardous environment which makes it unwise to perform the other activity at the same time?
4. Would the performance of the activities at the same time require an unreasonable allocation of resources?

The determination of what constitutes an unreasonable allocation of resources depends on the professional judgement of the estimator. If, after asking these questions, it is determined that the activities perform all at the same time. If this situation is repeated many times in a project, there may be thousands of different network diagrams which could be drawn. One school of thought contends that by assuming an artificial sequence many scheduling possibilities may be overlooked, so that if it is technologically feasible for several activities to take place concurrently, they must be represented in this way on the network diagram. It is possible that the computer analysis may reveal that there is sufficient time to perform the activities in any given sequence.

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The network diagrams representing the sub-projects must be meshed to form the overall project network. The physical connection is by an activity called an "interface dummy." The interface dummy originates or terminates at a node in the diagram (see Fig. 12), its other terminus being represented by a wavy line. It is considered to belong to the network from which it originates.

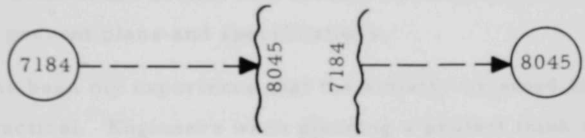

Fig. 12. Interfacial Dummy Activity

The relationship between sub-projects is determined only by technological and environmental limitations. In practice, the sub-projects are quite specialized and require their own peculiar equipment and apparatus. The common resource is manpower.

Once the master network has been assembled, it is analyzed by a computer and projected manpower requirements obtained. If manning levels have been exceeded, activities or sub-projects may have to be displaced in time, for which purpose criticality may be used as an index of flexibility. This will be discussed in a later section.

The preceding discussion of network preparation has been activity oriented. This is the philosophy of the Critical Path Method. PERT places greater emphasis on the concept of an event. The PERT definition of an event is "A meaningful specified accomplishment (physical or intellectual) in the program plan, recognizable as a particular instant in time. Events do not consume time or resources..." This orientation is natural since the designers of PERT wanted it to be can occur concurrently, they are shown occurring concurrently on the network diagram. This is the method used for inter-project planning.

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This orientation is natural since the designers of PERT wanted it to be compatible with the Navy's already existing Milestone Reporting Technique. The events on the original networks were identified with the previously defined milestones. Activities are considered as time- and resource-consuming elements which link events.

Personnel preparing PERT charts are instructed to:

1. Prepare a list of significant events.
2. Place the events on a flow chart by drawing a series of articles or boxes with the event description written inside.
3. Connect the events with arrows in accordance with present plans and specifications.

It has been my experience that the activity-oriented diagram is the most practical. Engineers when planning a project think in terms of operations. It is simpler to recognize the relationships between activities than between events. The relationships are depicted originally, not added at the end, when some of them may be overlooked. The data necessary to evaluate the network are of necessity activity oriented, i.e., duration times, man power and machine requirements, and money.

The diagrams representing the ZGS project are all in terms of activities. No labels are attached to the nodes except a number which serves to identify the activity. The concept of an event is not completely abandoned. There is provision in the computer program to obtain special print-outs on certain, selected, major milestones. This serves as a summary report to the higher levels of management.

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CHAPTER IV
NUMBERING THE NETWORK DIAGRAM

Once the network diagram has been constructed, it is necessary to label each event or activity. The labels are of two kinds: (1) one that a person examining the diagram can understand, and (2) one that a computer can understand. The first, of course, is simply a written name for the activity placed immediately above the arrow representing it, or in the case of PERT inside the circle representing the event. The second is usually in the form of a number or reducible to a number.

It is the node or event which is numbered. An activity is identified by a pair of numbers, those of its tail and head nodes. Thus, the activity in Fig. 13 would be referred to as 795-802.

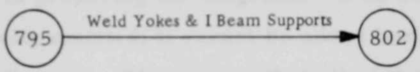

Fig. 13. Numbered Activity

There are three basic systems for assigning numbers to the diagram:

1. Sequential - no missing numbers
2. Sequential - missing numbers
3. Random

Sequential - No Missing Numbers

Numbers are assigned beginning from zero or one. The tail number of an activity must be less than the head number, and no numbers may be omitted.

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The disadvantage of this system is that it leaves very little margin for error in preparing the diagrams. If an essential activity has been accidentally omitted, it cannot be added later without remembering the entire diagram. This means that all of the punched cards used as computer input must be changed.

The sequential - no missing number system has been used by programs written for small or medium-sized computers. One example is LESS (Least Cost Estimating and Scheduling) written for the IBM 650. Some later variations of LESS do not use this system.

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Using this system, it is possible to insert or delete activities from a network without altering the entire network. When preparing the network, the planner can omit numbers in areas of uncertainty so that activities can be added at a later date.

Node numbers must be assigned storage locations in the computer, thus increasing the amount of storage required.

Before a network can be evaluated to determine the earliest and latest activity or event times, it is necessary to arrange the activities in a predecessor-successor order. Considering the activities as being arranged in a list, no activity can appear on the list until all of the activities preceding it have appeared. With a sequential numbering system if the activities are in numerical order they are also in successor-predecessor order. It is also possible to determine rapidly if an activity has been mistakenly represented as its own predecessor. In Fig. 14, activity C is immediately identified as its own predecessor because its tail node number, 10, exceeds its head node number, 4, in violation of the numbering rule.

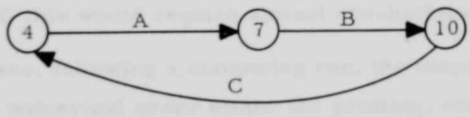

Fig. 14. Detection of Loop Error

The disadvantage of this system is that it is always possible for the planner to leave out an insufficient number of numbers. The computer program written to process the ZGS data uses the sequential - missing number method. We have not had to perform any extensive renumbering of our networks.

Random Numbering

Numbers can be assigned in a completely random manner. This method offers the greatest flexibility in the preparation of the network diagram.

A PERT number, for example, consists of a six-digit prefix and a three-digit suffix. The prefix identifies the program, company, and highest level diagram upon which the event appears. The suffix is assigned sequentially to the events as they are placed upon the diagram.

All PERT or CPM variations use punched cards to store activity information. To make this information accessible to clerical personnel, it is filed numerically by activity or event number. As stated earlier, before the computer can perform the network calculations the data must be sorted into a predecessor-successor order, which in most cases involves a completely separate computer run. The NORC computer, when used for processing PERT networks, requires nine minutes to sequence one thousand events. Three thousand events or activities is not an unreasonable figure to process at one time. In the case of NORC this would require almost one-half hour.

In all cases, following a computing run, the output must be sorted back into numerical order either for printing, editing, or merging with other data, such as an alpha-numeric job description. With a sequential system this sort is not needed.

A novel approach to node identification has been taken by the Philco Company in their program for the Philco 210 computer, called Philco PERT. The node label need not be a number at all, but may be any combination of not more than nine alpha-numeric characters. This label also constitutes a verbal description of the event (node). This eliminates the necessity of merging job descriptions with the output.

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This approach, although appealing from a data-processing standpoint, is not very practical, since it is very frequently impossible to describe an event adequately using only nine letters.

The gain in flexibility obtained by using random numbering would in most cases be off-set by the added cost of data processing. For most applications the sequential - missing number method, assuming a judicious assignment of numbers, would seem the most practical.

CHAPTER V
ACTIVITY DURATION TIMES

After a network diagram has been prepared and checked for accuracy, the next step in the analysis is to estimate the amount of time that will be required to perform each activity.

Rarely will the estimator be able to predict the exact amount of time required for the activity. The estimate will be a most likely duration, which is the most probable value of an unknown duration distribution. If the variance of this distribution is small, then the duration is approximately deterministic; if it is large, the duration is nondeterministic.

By making a detailed analysis it may be possible to reduce a nondeterministic case to a less nondeterministic one. As an example, consider an activity which is estimated to require one hundred days, and assume that the probability distribution has a standard deviation σ of ten days. Suppose that it is possible to break this activity up into ten ten-day activities with σ_i equal two days. Since $\sigma^2 = \sum_{i=1}^{10} \sigma_i^2$, the new standard deviation is $(40)^{1/2}$ or approximately six days; thus, the uncertainty of the estimate has been reduced.

It is almost never possible to determine the form of the activity distribution, let alone its variance, from the data available in the planning stages of a project. In practice, there are two alternatives: (1) assume a deterministic case, or (2) assume the form of the probability distribution and attempt to estimate its parameters. The originators of the Critical Path Method chose the first alternative; the originators of PERT, the second.

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The mean or expected value of the duration is used instead of the estimator's most likely duration. PERT assumes that in most cases the distribution will be asymmetrical, with the expected value falling between the most likely (mode) and the pessimistic times. The expected value $E(x)$ here is used in its statistical sense, i.e., for a continuous distribution of range (a, b), $E(x) = \int_a^b xP(x)\,dx$. There is a fifty percent probability that the expected value will be exceeded by the actual duration. The justification for this assumption is the observation that most likely times are more often exceeded than not.

The distribution, chosen on intuitive grounds, is the beta distribution:

$$f(t) = B(p+1,\ q+1)^{-1}\ (t-a)^p\ (b-t)^q/(b-a)^{p+q} \quad ,$$

where "a" and "b" are the optimistic and pessimistic times, and $B(p+1, q+1)$ is the beta function $\frac{\Gamma(p+1)\Gamma(q+1)}{\Gamma(p+q+2)}$. Using the transformations

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$$g(x) = B(p+1, q+1)^{-1} x^p (1-x)^q$$

The mode, expected value, and variance of this distribution are

$$X = \frac{p}{p+q}$$

$$E(x) = \frac{p+1}{p+q+2}$$

$$V(x) = \frac{(p+1)(q+1)}{(p+q+2)^2\,(p+q+3)} \quad .$$

Using the assumption that $VAR(x) \doteq (1/6)^2$, it can be shown that

$$(1) \qquad p^3 + (36X^3 - 36X^2 - 7X)\,p^2 - 20X^2 p - 24X^3 = 0 \quad .$$

If the three time estimates a, T, and b are given, X can be calculated from

$$X = \frac{T - a}{b - a} \quad .$$

Then X can be substituted into the above equation to solve for p and q with the aid of the equation

$$q = p\left(\frac{1}{X} - 1\right) \quad .$$

Plotting E(x) versus x reveals that E(x) can be approximated by the linear function $\frac{4X + 1}{6}$, so that $E(t) = f(a, T, b)$ can be approximated by $\frac{a + 4T + b}{6}$.

This last equation is the one used in the PERT analysis to determine the expected activity duration as a function of the optimistic, most likely, and pessimistic times.

The expected duration can be rewritten as

$$E(t) = \frac{1}{3}\left(2T + \frac{a+b}{2}\right) \quad ,$$

showing that the value lies one-third of the way from the mode to the mid range.

A good deal of the controversy concerning the PERT system centers around the three-time-estimate concept. The choice of the beta distribution does not rest on mathematical or solid experimental evidence, but rather is based on the assumption that estimated times are more often than not exceeded. It is questionable whether this excess is the result of uncertainty or of oversight. The concept of a pessimistic and optimistic time have not been defined as clearly as that of a most probable time. Two technically qualified estimators would very likely give quite similar estimates of the most probable time required to perform an activity but could differ considerably in their estimates of the optimistic and pessimistic times.

The effect of using the three time estimates is to give a more pessimistic outlook than would be obtained from using a single estimate. A study by W. W. Haase of NASA[1] indicates that the degree of pessimism is proportional to the expected date of accomplishment of the event in question. A straight line fitted to Mr. Haase's data results in an equation of the form

$$P \doteq 0.25T \quad ,$$

where P is the pessimism, in weeks, added to the prediction by using three estimates instead of one, and T is the predicted time to the accomplishment of the event, expressed in months.

As has been said many times, the real value of network techniques lies in the detail planning required early in the project to construct the diagrams. This benefit is obtained with either one or

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three time estimates. The primary purpose of the network analysis is to uncover the "critical path," the chain of activity which determines the duration of a project. The activities normally constitute a few percent of the total number. The experience of some users indicates that, even if the initial duration estimates are inaccurate by twenty percent, this is close enough to prevent the danger of confusing the critical path with other activity sequences.[1] Refinement of the analysis is obtained by carefully reexamining the estimates of those activities which are critical or have small amounts of slack. A certain amount of error can be tolerated in the other estimates, since these activities have larger amounts of slack which can be used to absorb duration increases.

The purpose of programming the computation for an electronic computer is that it permits rapid evaluation of the effect of activity changes. Most installations update their calculations at least every two weeks. Thus, trends which may interfere with the project schedule can be recognized before they become acute.

The network analysis of the ZGS project adopted the Critical Path Method concept of a single estimate of duration time. To reduce the uncertainty, estimators were asked not to specify activities whose duration exceeds six weeks. If possible, these activities were to be broken up into several smaller ones. So as not to overburden the system with myriads of short, minor activities, the lower level of duration estimate was set at one day.

The unit of duration time varies from system to system. PERT commonly uses a week, with estimates expressible to tenths of a week. I have chosen the working day as the basic unit. Estimates are rounded off to the nearest integer value. Use of the working day permits the

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effect of weekends and holidays to be taken into consideration and permits easy conversion to a calendar date. I feel that the use of integer values is justified, considering the probable accuracy of the data. It also has the advantage of reducing the amount of computer time necessary to perform the calculation. The time saved is not large and should not be a primary consideration in writing a program.

The merits of three time estimates versus one have not been resolved at the time of writing this paper. The Navy and the Air Force, which uses the Navy's PERT system, still require the submission of three estimates. The Air Force discourages "odd-ball" PERT-like systems,[1] which use a single estimate. The National Aeronautics and Space Administration when making PERT analyses on the Navy's IBM 7090 computer at Dahlgren, Virginia, by means of a standard Navy PERT program, requires that all three time estimates be the same. This is, in effect, a single estimate, since the program uses the formula

$$E(t) = \frac{T + 4T + T}{6} = \frac{6T}{6} = T \quad ,$$

to compute the expected time. NASA is in the process of developing its own program which will use one estimate. Some "canned" computer programs have provision for either system, the option being under sense switch control. In general, three-time-estimate systems seem to be favored by the military and single-time-estimate systems by private industry.

[1] AFSC Policies & Procedures Handbook, Aeronautical Systems Division of Air Force Systems Command, January 1962. p. IX-20.

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CHAPTER VI
MATHEMATICAL EVALUATION OF THE NETWORK

The next step in the analysis after the diagram has been numbered and duration times assigned is to determine the time boundaries between which each activity must be performed.

Each activity is designated by two numbers, "i" and "j," the tail and head node numbers, respectively. The duration of each job is d_{ij}. It is assumed that all d_{ij} are deterministic.

<u>Computation of Earliest Starting Times</u>

Consider a network containing N nodes. Define the earliest time that any activity leaving node j can start as t_j. Diagramming rule number one[1] states that no activity leaving a node can begin until <u>all</u> activities entering the node have been completed. As a consequence $t_j = \text{MAX}\ (t_i + d_{ij})$.

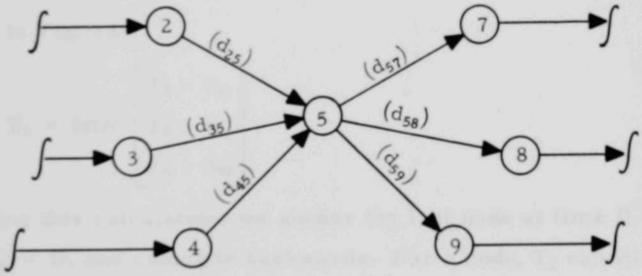

Fig. 15. Sample Network

In the isolated portion of a network shown in Fig. 15 it is desired to determine the earliest time that any activity leaving node number five can start. The condition is

$$t_5 = \text{MAX} \begin{bmatrix} t_2 + d_{25} \\ t_3 + d_{35} \\ t_4 + d_{45} \end{bmatrix} \text{ where } i = 2,3,4.$$

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[1] p. 11.

By definition, t_1, the earliest time for the first node in the diagram (no activities enter node number one), is zero. The earliest time that node N, the last node, can be reached is D, the expected duration of the project.

Before we can calculate t_j for any node, we must know t_i for all nodes immediately preceding it. Thus we must start at the beginning of the diagram and work through to the end. This is referred to as making a "forward pass" through the network.

Computation of Latest Finishing Times

The next quantity to be calculated is the latest time by which each node must be reached so as not to prolong the overall duration of the project. This quantity, called the latest finishing time, is represented symbolically as T_i:

$$T_i = \text{MIN}\,(T_j - d_{ij})$$

In Fig. 15,

$$T_5 = \text{MIN}\begin{bmatrix} T_7 - d_{57} \\ T_8 - d_{58} \\ T_9 - d_{59} \end{bmatrix} .$$

In making this calculation, we anchor the last node at time D, i.e., $T_n = t_n = D$, and calculate backwards. For a node, T_i cannot be calculated until T_j for all nodes immediately following it have been determined. This is referred to as making a "backward pass" through the network.

Sample Calculation

To illustrate the calculation consider the network in Fig. 16. The values of the earliest starting time t_i are entered in the circles, the values of latest finishing time T_i, in the squares.

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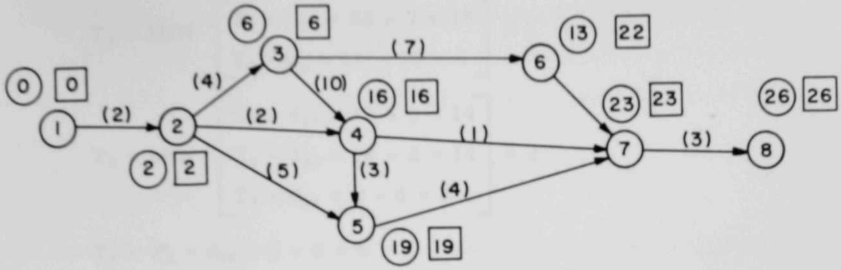

Fig. 16. Sample Network

Forward Pass

$t_1 = 0$ (by definition).

$t_2 = t_1 + d_{12} = 0 + 2 = 2$

$t_3 = t_2 + d_{23} = 2 + 4 = 6$

$$t_4 = \text{MAX}\begin{bmatrix} t_2 + d_{24} = 2 + 2 = 4 \\ t_3 + d_{34} = 6 + 10 = 16 \end{bmatrix} = 16$$

$$t_5 = \text{MAX}\begin{bmatrix} t_2 + d_{25} = 2 + 5 = 7 \\ t_4 + d_{45} = 16 + 3 = 19 \end{bmatrix} = 19$$

$t_6 = t_3 + d_{36} = 6 + 7 = 13$

$$t_7 = \text{MAX}\begin{bmatrix} t_4 + d_{47} = 16 + 1 = 17 \\ t_5 + d_{57} = 19 + 4 = 23 \\ t_6 + d_{67} = 13 + 1 = 14 \end{bmatrix} = 23$$

$t_8 = t_7 + d_{78} = 23 + 3 = 26 = D$

Backward Pass

$T_8 = t_8 = D = 26$ (by definition)

$T_7 = T_8 - d_{78} = 26 - 3 = 23$

$T_6 = T_7 - d_{67} = 23 - 1 = 22$

$T_5 = T_7 - d_{57} = 23 - 4 = 19$

$$T_4 = \text{MIN}\begin{bmatrix} T_7 - d_{48} = 23 - 1 = 22 \\ T_5 - d_{45} = 19 - 3 = 16 \end{bmatrix} = 16$$

$$T_3 = \text{MIN} \begin{bmatrix} T_6 - d_{36} = 22 - 7 = 15 \\ T_4 - d_{34} = 16 - 10 = 6 \end{bmatrix} = 6$$

$$T_2 = \text{MIN} \begin{bmatrix} T_5 - d_{25} = 19 - 5 = 14 \\ T_4 - d_{24} = 16 - 2 = 14 \\ T_3 - d_{23} = 6 - 4 = 2 \end{bmatrix} = 2$$

$$T_1 = T_2 - d_{12} = 2 - 2 = 0$$

Inherent Bias

If all of the duration times were, as assumed, accurately known, there would be no objection to the above procedure. However, because the duration times are selected from an unknown distribution, there is an inherent bias which biases all earliest starting times towards the start of the project. To gain an insight into the nature of this bias, consider two activities whose duration distributions are normal and have the same mean and standard deviation. Also, assume that each activity starts at the same time.

If the activity durations are the expected value of their probability density distributions, what is the expected value of t_2?

Define event number one, E_1, as the event that the first activity is completed in time $\leq d_{ij}$, and E_2 as the event that the second activity is completed in time $\leq d_{ij}$. The event that node number two is reached in time $\leq d_{ij}$ is E_3. Since each activity is assumed to be independent of the other,

$$P(E_3) = P(E_1)P(E_2) = (1/2)(1/2) = 1/4 \quad .$$

In other words, there are three chances out of four that any activity leaving node two cannot start until some time in excess of d_{ij}. The expected value of t_2 is that for which $P(t_2) = 1/2$, so that

$$P(E_1') = P(E_2') = (1/2)^{\frac{1}{2}} = 0.707 \quad .$$

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From a table of values of the standard normal distribution function, E_1' and E_2' are the events that the duration of each activity does not exceed $d_{ij} + 0.55\sigma$, where σ is the standard deviation of the distribution, so that

$$E(t_2) = d_{ij} + 0.55\,\sigma \quad .$$

The amount $0.55\,\sigma$ would be ignored in the described method of computation. The case illustrated is the worst possible. As the difference in duration between the two events increases, the expected value of t_2 approaches more closely the expected value of d_{ij}, but can never be less than d_{ij}.

The PERT analysis attempts to obtain a measure of the variance of the earliest and latest times by assigning to the node the expected value and variance of the longest path leading in to it. Even though the individual activity durations are assumed to fit a beta distribution, the earliest and latest times are assumed to be normally distributed. The justification for this assumption is the Central Limit Theorem. The probability of meeting a particular schedule date can be calculated by means of a normal distribution table.

Having calculated the earliest and latest times, we can proceed to schedule the project by determining the earliest and latest starting and finishing times for each activity. We define the following quantities:

1. Earliest starting time for job (i, j)
   $= (JEST)_{ij} = t_i$
2. Earliest finishing time for job (i, j)
   $= (JEFT)_{ij} = t_i + d_{ij}$
3. Latest finishing time for job (i, j)
   $= (JLFT)_{ij} = T_j$
4. Latest starting time for job (i, j)
   $= (JLST)_{ij} = T_j - d_{ij}$

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3. Latest finishing time for job (i,j)
   $= (LFT)_{ij} = T_j$
4. Latest starting time for job (i,j)
   $= (LST)_{ij} = T_j - d_{ij}$

The latest starting and finishing times are interpreted as the times by which any activity must be started or finished so as not to prolong the duration of the project, assuming that subsequent activities are not expedited.

The Critical Path

The maximum time available to perform activity (i, j) is $T_j - t_i$, where $T_j - t_i \geq d_{ij}$. If $T_j - t_i = d_{ij}$, the time available to perform the activity equals the time required to perform the activity. The activity is said to be "critical." If the scheduled completion date of the project is equal to the calculated project duration, D, there will be at least one, unbroken chain of critical activities connecting the beginning and end of the project. This chain of critical activities, called the "critical path," determines the project duration.

Slack

A measure of the criticality of each activity can be obtained by taking the difference between the time available and the duration: $T_j - t_i - d_{ij}$. This amount of time is called "total slack" or "total float." Total slack can be expressed in several different, but equivalent, ways:

1. $(TS)_{ij} = T_j - t_i - d_{ij} = (JLFT)_{ij} - (JEST)_{ij} - d_{ij}$
2. $(TS)_{ij} = T_j - (t_i + d_{ij}) = (JLFT)_{ij} - (JEFT)_{ij}$
3. $(TS)_{ij} = (T_j - d_{ij}) - t_i = (JLST)_{i,j} - (JEST)_{ij}$.

The total slack is the amount of time that the start or finish of an activity can be delayed and still not prolong the duration of the project. If the total slack of job (i, j) is used up, then jobs leaving node j can no longer start at the earliest possible time. This leads to the question: how long can the start or completion of job (i, j) be

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$$(JEFT)_{ij} + (FS)_{ij} = (JEST)_{jk}$$

$$\begin{aligned}(FS)_{ij} &= (JEST)_{jk} - (JEFT)_{ij} \\ &= t_j - t_i - d_{ij}\end{aligned}$$

If job (i, j) is the only one entering node j, then $(FS)_{ij} = 0$.

The difference between the latest finishing time of a job entering node j and the earliest starting time of a job leaving node j is defined as "dependent slack."

$$\begin{aligned}(DS)_{ij} &= (JLFT)_{ij} - (JEST)_{jk} \\ &= T_j - t_j \\ &= T_j - t_i - d_{ij} - (t_j - t_i - d_{ij}) \\ &= (TS)_{ij} - (FS)_{ij}\end{aligned}$$

If activity duration times are not changed, $(JLFT)_{ij}$ is a constant. Therefore, if some of $(DS)_{ij}$ is used up, $(JEST)_{jk}$ must increase. Thus, if dependent slack is used up, the project can still be completed by the planned date, but not all activities can start at their originally planned date. This quantity is also called "interfering slack" since its consumption interferes with the earliest starting times of some activities.

We may also ask: how much time is available between the earliest starting time of a job leaving node j and the latest finishing time of a job entering node i? This quantity minus the activity duration is "independent slack." An activity can be displaced by this amount without affecting any other activity.

$$\begin{aligned}(IS)_{ij} &= (JEST)_{jk} - (JLFT)_{hi} - d_{ij} \\ &= t_j - T_i - d_{ij} \quad .\end{aligned}$$

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$$(IS)_{ij} = (JEST)_{jk} - (JLFT)_{hi} - d_{ij}$$

$$= t_j - T_i - d_{ij}$$

This quantity may be negative, so that the definition must be modified to the form

$$(IS)_{ij} = MAX\ (t_j - T_i - d_{ij},\ 0) \quad .$$

Using the definitions presented, we can calculate the activity schedule for the sample network as in Table 1.

Table 1. Sample Activity Schedule

Job Number		Duration	Starting Time		Finish Time		
i	j	d_{ij}	JEST	JLST	JEFT	JLFT	Slack
1	2	2	0	0	2	2	0
2	3	4	2	2	6	6	0
2	4	2	2	14	4	16	12
2	5	5	2	14	7	19	12
3	4	10	6	6	16	16	0
3	6	7	6	15	13	22	9
4	5	3	16	16	19	19	0
4	7	1	16	22	17	23	6
5	7	4	19	19	23	23	0
6	7	1	13	22	14	23	9
7	8	3	23	23	26	26	0

The critical path is: (1,2) - (2,3) - (3,4) - (4,5) - (5,7) - (7,8). If any of these activities is allowed to slip without a corresponding expediting of another activity, the project will be prolonged past the twenty-sixth day.

Project managers can use slack to indicate possible resource trade-offs from slack to critical activities. The proper use of slack permits the project manager to exercise management by exception. The higher levels of management need only concern themselves with

the critical or near-critical activities, leaving the lower levels to insure that the non-critical activities are accomplished at the proper time.

From Table 1 it can be seen that six out of the eleven jobs are critical. In larger networks for real projects, less than ten percent of the jobs are normally critical; the larger the network the smaller is the percentage.

It should be noted that slack is a function of the activity duration and the path on which the activity lies. Using up some of the slack by delaying the start or finish of an activity will remove some slack from some of the "downstream" activities.

If the computed completion date of an activity in the network is greater than the scheduled completion date, the finish date can arbitrarily be fixed at the scheduled date and a backward pass made from that point. Some paths will be revealed to have negative total slack. The negative slack indicates by how many units of time the path in question must be shortened in order to meet the scheduled date.

The type of slack so far developed have been activity slacks, i.e., the slack associated with an activity. Event-oriented PERT systems calculate "event slack," which is defined as the difference between the latest finishing time and earliest starting time for a given node (event):

$$(ES)_i = T_i - t_i \quad .$$

The critical path is the chain of activities connecting events with $(ES)_i = 0$.

To prove that the same critical path is obtained in both cases, consider a network whose critical path is known. All of the total activity slacks along this path are equal to zero by definition, so that the critical or near-critical activities, leaving the lower levels to insure that the non-critical activities are accomplished at the proper time.

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1. $T_n - t_{n-1} - d_{n-1,n} = 0$

$T_{n-1} - t_{n-2} - d_{n-2,n-1} = 0$

$\cdot \quad \cdot \quad \cdot \quad \cdot$

$\cdot \quad \cdot \quad \cdot \quad \cdot$

$\cdot \quad \cdot \quad \cdot \quad \cdot$

$T_2 - t_1 - d_{1,2} = 0$

$T_1 - t_0 - d_{0,1} = 0$

Adding the equations we obtain

2. $$\sum_{i=1}^{n} T_i - \sum_{i=0}^{n-1} t_i - \sum_{i=1}^{n} d_{i-1,i} = 0$$

3. $$T_n + \sum_{i=1}^{n-1} T_i - \sum_{i=0}^{n-1} t_i - \sum_{i=1}^{n} d_{i-1,i} = 0$$

$\sum_{i=1}^{n} d_{i-1,i} = D$, since the critical path determines the projects duration.

$T_n = D$ by definition

4. $$\sum_{i=1}^{n-1} T_i - \sum_{i=0}^{n-1} t_i = 0$$

$t_0 = 0$ by definition

5. $$\sum_{i=1}^{n-1} T_i = \sum_{i=0}^{n-1} t_i$$

6. For any node $T_i \geq t_i$

7. Assume one of the $T_i > t_i$.

Then there is at least one set of (T_k, t_k) such that $T_k < t_k$, in order to satisfy equation five, since $T_{i+1} > T_i$ and $t_{i+1} > t_i$.

This violates statement six; therefore statement seven cannot be true and all $T_i = t_i$. All event slacks along this path are zero.

CHAPTER VII
APPLICATION OF NETWORK ANALYSIS TECHNIQUES TO THE ZERO GRADIENT SYNCHROTRON PROJECT

The Zero Gradient Proton Synchrotron

The Zero Gradient Synchrotron (ZGS) is under construction at the Argonne National Laboratory, located twenty-five miles southwest of downtown Chicago. When completed, this forty-seven million dollar particle accelerator will be one of the world's largest, possessing some capabilities found in no other.

The ZGS will consist of a four-thousand-ton ring of magnets, two hundred feet in diameter. This ring consists of eight sections of magnet blocks separated by straight metal sections which contain vacuum pumps and control apparatus. Encased within the magnet blocks will be a thin, spacemetal, evacuated tube, known as the inner vacuum chamber. Every four seconds pulses of up to 10^{13} protons can be accelerated around the ring through this chamber at speeds approaching that of light.

In operation, the protons will be pre-accelerated to eight hundred thousand electron volts by a Cockcroft-Walton voltage multiplier. From there, they will pass into a linear accelerator in which they will be accelerated to fifty million electron volts before being introduced into the magnet ring. While in the ring, they will travel more than seven times the distance around the earth while being accelerated to an energy of twelve and one half billion electron volts.

To hold the particles in a nearly circular orbit during acceleration, the magnetic field within the ring is carefully synchronized with the frequency of the accelerating radiofrequency cavity. During a one-second pulse, the magnetic field varies from zero to twenty-four thousand gauss. The energy for this field is provided by electric current

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Parts of the ZGS will be covered with earth to absorb the penetrating radiations produced when it is in operation. The "Ring Building," a doughnut-shaped concrete structure, two hundred and ten feet in diameter and fifty eight feet high, which houses the ring magnet, will be covered with fifty feet of earth.

The "Center Building," a ninety-foot structure built through the center of the Ring Building, houses the equipment which rectifies and controls this current. During a one-second pulse, the current varies from zero to eleven thousand amperes, at twelve thousand five hundred volts. In order that the power demand may be smoothed out, current from the line passes into a motor-generator system external to the ring. The motor-generators drive a sixty-eight-ton steel flywheel, thirteen feet in diameter, at nine hundred revolutions per minute.

The ZGS has been designed to produce all thirty presently known or anticipated subatomic-particles in larger quantities than any other accelerator. The final energy of twelve and one-half billion electron volts is just above the threshold energy necessary to produce antiprotons. The particles are produced either within the machine itself or by the action of an external beam. The original design calls for three beam extraction areas, two proton areas, and one meson area. At present, only two of the areas, one proton and the meson, are being developed.

The structures of the ZGS complex will cover forty-seven acres. Construction of a high-energy physics-research center, valued at nearly seven million dollars, will begin in the spring of 1962. This center will be adjacent to the ZGS.

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The ZGS will be made available to scientists from middlewestern universities. It represents a research facility which no university could hope to build and operate with its own funds. The completion of the ZGS will make the middlewestern United States one of the world centers of high-energy physics research.

Preliminary Studies

The possibility of using network techniques to plan and schedule some aspects of the synchrotron construction was first considered in April 1961. At this time, completion of the machine was scheduled for the summer of 1962. It was becoming evident that this date could not be attained, partially due to unexpected difficulties that were being encountered in the fabrication of the ring-magnet blocks. Because of this delay, it was apparent that the assembly operations which followed the delivery of the magnet blocks had to be planned with great care in order to minimize extension of construction beyond the scheduled completion date.

The responsibility for most of these assembly operations rested with the Mechanical Engineering Group of the Laboratory's Particle Accelerator Division. Mr. Willard Hanson of the Mechanical Engineering Group, who was investigating the possibility of using a critical-path-scheduling technique, asked the author to develop a PERT-like system to coordinate the remaining mechanical-engineering effort necessary to close the ring magnet.

It was envisioned that the final assembly would involve a large number of activities. To gain experience in preparing network diagrams, a smaller sub-project, the fabrication of the inner vacuum chamber, was chosen as the first study. About two hundred activities were involved. Some difficulty was encountered with the diagram, partly because of lack of familiarity with the project on the part of the author and partly because of lack of familiarity with diagramming on the part of

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Following the PERT philosophy, the estimator was asked to supply three time estimates, in working days, of activity duration. In the majority of the cases the most likely estimate was less than fifteen working days and the implied beta distributions almost symmetrical. In these cases the difference between the most likely and expected durations was usually less than one working day. At this point it was decided to use a single time estimate.

During the following weeks, diagrams were prepared depicting fabrication of the sidewalls of the outer vacuum chamber and of bending magnets 101 and 102; inspection, preparation, magnetic testing, and routing of the ring-magnet blocks from the time of receipt to installation in the ring; and final assembly of the ring. The method of data collection was inefficient. Due to the embryonic nature of the project (network analysis project), there were no administrative data flow systems devised. Information gathering usually consisted of the author "buttonholing" the estimator in his office between conferences or meetings.

While the original data were being collected, attention was also given to a method of processing it. The Laboratory possesses several computers: a small Recomp II with a magnetic disc memory; a medium-sized IBM 1620, IBM 1401, and GEORGE, an Argonne-designed computer of high speed; and in the large-size category, an IBM 704 with a thirty-two-thousand-word magnetic-core memory. Since the analysis is essentially a data-processing problem as distinguished from a computational problem, the characteristics desired were large the estimators. The diagram had to be redrawn several times, because, although it would correctly display activity relationships as determined by the estimators, close examination would indicate other relationships neither stated nor implied. With practice in the use of dummy activities, this problem was solved.

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The first computer program was written during the summer of 1961. In September, the first computer run, using thirty-five hundred activities, was made. This run took over one half hour, longer than had been expected. Re-examination of the program logic showed several places where a considerable amount of time could be saved.

In November, two additional changes were made in the program. The original program computed free slack; however, experience indicated that it was not nearly as useful as the total slack, which was also computed. The computation of free slack was eliminated. The original program identified an activity only by its head and tail number. Provision was made at this time to include a fifty-five-character alpha-numeric activity description as part of the printout.

The program currently in use is the third revision, made during January and February 1962. Provision was made to insert arbitrary times of activity completion, make a backward pass, and compute negative slack. Three buffer areas were set up in the magnetic-core memory to speed up the magnetic tape input-output. Finally, provision was made to obtain an optional event-oriented printout of significant project milestones.

Concurrent with the development of the basic program for critical path was the development of two other programs. The first of these computes man-power requirements, as a function of calendar date, for fifteen skill categories and edits the critical-path program output to produce special reports. The inputs to this program are two of the output tapes from the main program. The third program is a data checker. Occasionally during the card-to-tape run, either the first few or last few cards would not be written on tape. The storage capacity, rapid and flexible input-output devices, and high-speed off-line, auxiliary equipment. Of the computers available, the IBM 704 most nearly satisfied these requirements and was chosen.

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Data Organization

A rough sketch of the proposed network diagram is obtained from the estimator. A smooth copy is drawn on vellum, checking to insure that the basic diagramming rules have been obeyed. An Ozalid copy of the smooth diagram is then returned to the estimator for inspection and estimation of activity durations (in working days) and manpower requirements. Upon return of this copy, the estimates are transferred to the vellum master copy and the master copy is numbered.

The numbering system used is the sequential, numbers-missing, system. For most of the diagrams, even numbers have been used with a ten-number gap every twenty nodes or so. The numbers can be anywhere from one to five digits, with a maximum value of 32,767. A block of numbers is assigned to each sub-project.

The number blocks currently being used are:

0-1,999;	fabrication and testing of ring-magnet block
3,000-3,499;	fabrication of inner vacuum chamber
3,500-3,999;	fabrication of sidewalls of outer vacuum chamber
4,000-4,399;	fabrication and testing of bending magnets 101 and 102
4,400-4,999;	fabrication and testing of achromatic bending magnets, one, two, and three

6,000-6,999;	fabrication and testing of ring-magnet coils
7,000-8,999;	assembly of ring-magnet octants
10,000-10,299;	fabrication and installation of beam-detection apparatus
11,000-11,100;	construction of meson wall
11,500-12,299;	fabrication, installation, and testing of straight-section vacuum boxes.

New Ozalid copies are prepared of the numbered diagram and, after a final check by the estimator, approved for release.

A five-inch-by-eight-inch card is prepared for each non-zero activity on the diagram (see Figs. 17 and 18). This card serves as a chronological history of the activity. As changes in estimates are made, they are recorded on the card, along with the date and the name of the person making the estimate.

Job Number	Sub-project:
Prepared By: Date:	Job Description

Job Duration

Days	Date	Source	Days	Date	Source

Fig. 17. Activity File Card (Front)

MAN POWER REQUIREMENTS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Source	Date

Fig. 18. Activity File Card (Back)

Originally, the networks contained thirty-five hundred activities. Two thousand of these, many of which were dummies, were contained in the diagrams representing fabrication and testing of the ring-magnet block. After several computer runs, it was realized that the amount of information being obtained from this diagram did not justify such a high degree of detail. The diagram was reduced to five hundred activities. At the present time, the project diagrams consist of approximately two thousand activities, of which approximately four hundred are dummies.

The Critical Path Computer Program

The program is written in Fortran II for an IBM-704 with 32K memory, five tape units, and on-line card reader, printer, and punch.

Capacity is forty-five hundred activities with no restriction on the number of events. All arithmetic is integer. A word of storage in the 704 consists of three parts: address, tag, and decrement.

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Fortran-compiled programs store integers in the decrement portion. If the present capacity should become restrictive, the program has been designed so that the data can be conveniently packed to give capacities of 5,400; 6,750; or 9,000 activities, depending on the degree of packing used. Running time of the unpacked program is from four to seven minutes for two thousand activities, depending on the output options chosen. The basic computation exclusive of input-output is between ninety and one hundred and twenty seconds.

The basic documents are punched cards. There are four types of cards, kept in separate files. The first card is the activity data card (see Fig. 19), which is divided into nineteen fields. The first two fields contain i and j, the tail and head node numbers. Field three contains the activity duration. The next fifteen fields contain the manpower requirements. Field nineteen is for comments. These comments normally concern whether or not the activity has been expedited. The second card is the activity description card (see Fig. 20) and consists of three fields. Fields one and two again contain i and j. Field three contains a fifty-five-character alpha-numeric job description.

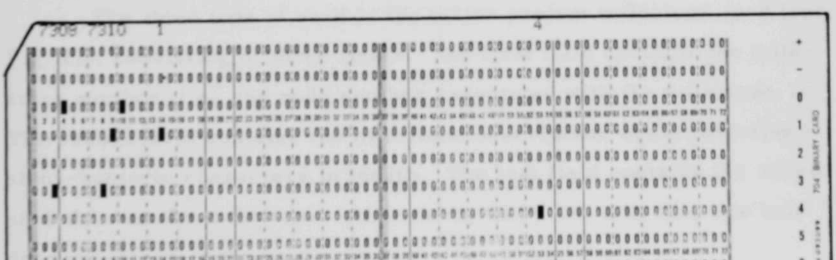

Fig. 19. Activity Data Card

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Fig. 19 Activity Data Card

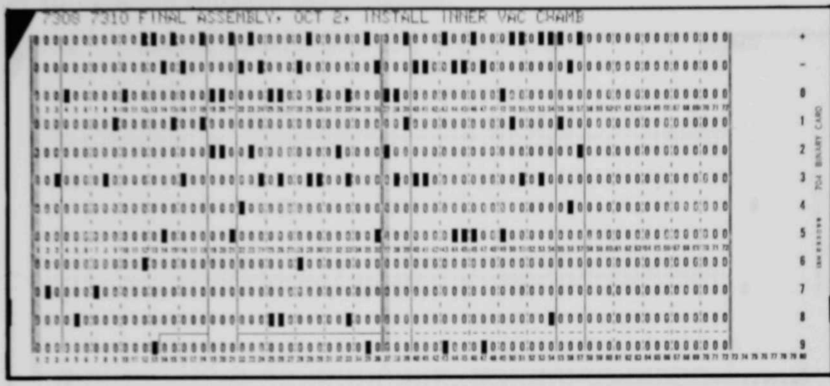

Fig. 20. Activity Description Card

An activity description consists of three parts:

1. sub-project description;
2. component number (if applicable); and
3. the operation or function.

There are no description cards for dummy activities. All cards are filed in j sequence within i sequence.

The third type of card is the major project milestone card (see Fig. 21), consisting of three fields. The first field contains the milestone number, i.e., the node number associated with the milestone. The second field contains the milestone description, again fifty-five alpha-numeric characters in length. The last field contains the scheduled date of the milestone. At present, there are less than one hundred milestone cards.

The fourth set of cards, calendar date cards (see Fig. 22), contains the calendar date of each working day of the project. The date is a six-digit number in the order month, day, and year. The use of

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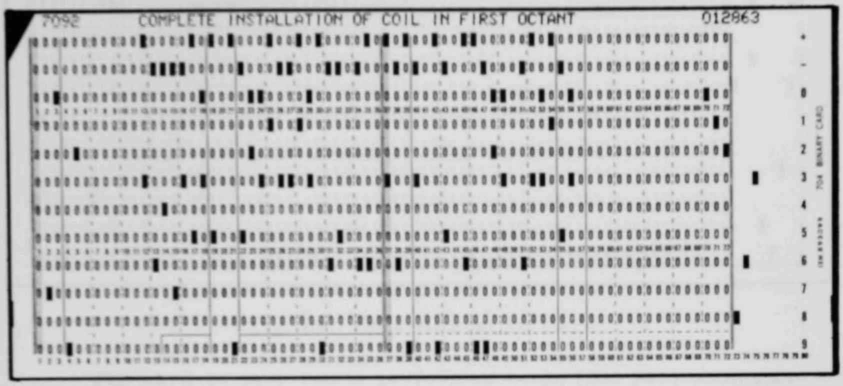

Fig. 21. Project Milestone Card

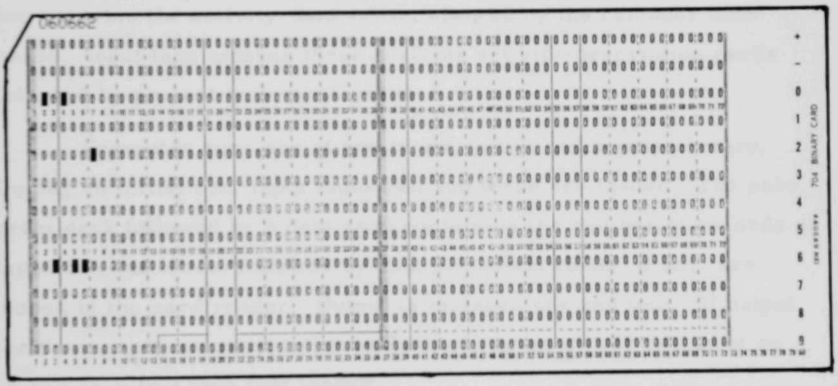

Fig. 22. Calendar Date Card

The fifth type of card, which is not filed, is the data-alteration card (Fig. 23). These are used to determine the effect of changes of activity duration or of arbitrary times of activity completion. Their format is identical with that of the activity-data card except that column seventy contains a control digit which determines how the program will manipulate the data on the card.

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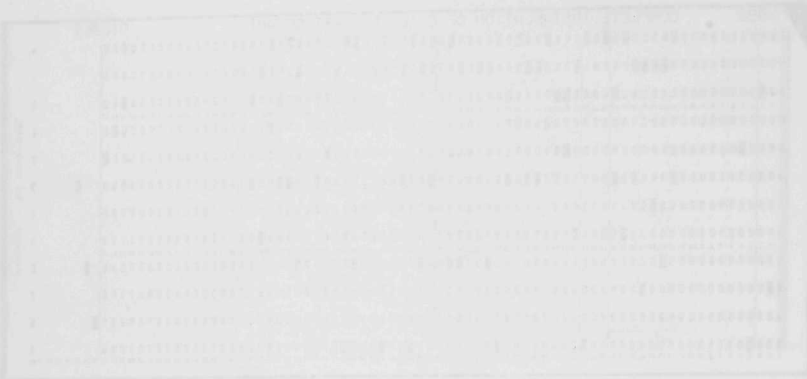

[illegible]

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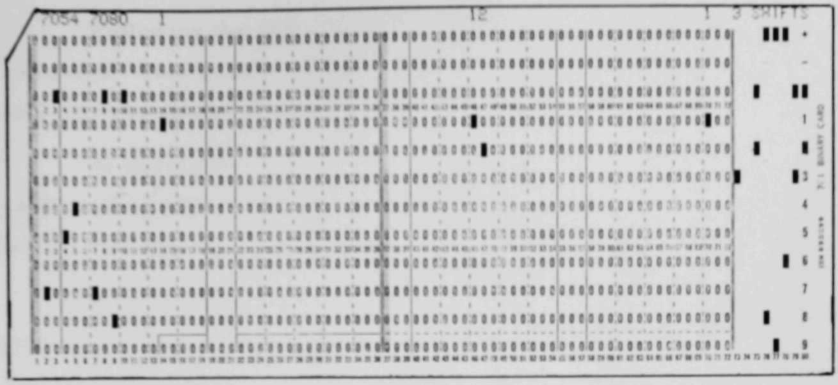

Fig. 23. Data Alteration Card

The card-to-tape run is performed by an IBM 1401 computer and is based on a standard Argonne script routine. Input tape number two contains the activity-data cards followed by the calendar date cards. Input tape number three contains activity-description cards followed by project-milestone cards.

Computer runs are of two types, initial and supplementary. During an initial run, input tapes two and three are loaded. The program deck followed by a data card containing the number of records on tapes two and three, followed by data alteration cards, if any, are placed in the card reader. Output is on tapes one and four. If output for the manpower computation is desired, sense switch two must be up and an output tape five loaded.

After the program has been loaded, the activity data cards and the calendar date cards are read in from tape two. Tape two is rewound. The data are checked for incorrect numbering, sequencing, and duplications. If there are errors, they are printed on-line and, following the completion of the data check, the program stops with an HPR22222. Computation cannot proceed until the data have been corrected.

Figure 2— Data Alteration Card

The card-to-tape run is performed by an IBM 1401 computer and is based on a standard Argonne script routine. Input tape number two contains the activity-date cards followed by the calendar date cards. Input tape number three contains activity description cards followed by project-milestone cards.

Computer runs are of two types, initial and supplementary. During an initial run, input tapes two and three are loaded. The program deck followed by a data card containing the number of records on tapes two and three, followed by data alteration cards, if any, are placed in the card reader. Output is on tapes one and four. If output for the manpower computation is desired, sense switch two must be up and an output tape five loaded.

After the program has been loaded, the activity-data cards and the calendar date cards are read in from tape two. Tape two is rewound. The data are checked for incorrect numbering, sequencing, and duplications. If there are errors, they are printed on-line and, following the completion of the data check, the program stops with an HPH22222. Computations cannot proceed until the data have been corrected.

Following the data check, an indexing parameter, JKMIN, is computed. This quantity determines the lower limit of indexing for each activity during the backward pass.

At this point, if sense switch three is down, data-alteration cards will be read from the card reader and the data in storage altered. The control digit, NQ, in column seventy controls the program.

If sense switch three is up or if NQ = 2, the forward pass begins. The earliest starting time for each activity, JEST, is computed. The maximum value of JEST is computed and all latest finishing times, JLFT, set equal to it.

The backward pass now begins and all JLFT are computed.

Following this, the output phase begins. The input buffer for tape three is loaded with ten records of activity-description data. If sense switch two is up, the input buffer for tape two is loaded with ten records of manpower data. The earliest finishing time, JEFT; latest starting time, JLST; and the total slack, MTFT, are computed. JEST, JEFT, JLST, and JLFT are converted to their equivalent calendar dates. These data merged with the appropriate activity description and manpower data are transferred to the output buffer.

Errors in the activity-description file will be printed on-line, under control of sense switch five. If the program cannot locate the proper activity description, zeros will be substituted and the program will proceed.

When the output buffer has been loaded with ten records, output on tape four takes place. If sense switch two is up, JEST, JEFT, and manpower data will be written on tape five.

Following completion of the first output phase, milestone output begins, under control of sense switch four.

Following the date check, an indexing parameter, IKMIN, is computed. This quantity determines the lower limit of indexing for each activity during the backward pass.

At this point, if sense switch three is down, date-alteration cards will be read from the card reader and the data in storage altered. The control digit, NQ, in column seventy controls the program.

If sense switch three is up or if NQ = 2, the forward pass begins. The earliest starting time for each activity, JEST, is computed. The maximum value of JEST is computed and all latest finishing times, JLFT, set equal to it.

The backward pass now begins and all JLFT are computed.

Following this, the output phase begins. The input buffer for tape three is loaded with ten records of activity-description data. If sense switch two is up, the input buffer for tape two is loaded with ten records of manpower data. The earliest finishing time, JEFT, latest starting time, JLST, and the total slack, MTFT, are computed. JEST, JEFT, JLST, and JLFT are converted to their equivalent calendar dates. These data merged with the appropriate activity description and manpower data are transferred to the output buffer.

Errors in the activity-description file will be printed on-line, under control of sense switch five. If the program cannot locate the proper activity description, zeros will be substituted and the program will proceed.

When the output buffer has been loaded with ten records, output on tape four takes place. If sense switch two is up, JEST, JEFT, and manpower data will be written on tape five.

Following completion of the first output phase, milestone output begins, under control of sense switch four.

Next, calendar dates are written on tape five. If the present value of NQ is five, the program will advance. If not, it will loop back and read more data-alteration cards. If NQ = 5, output tapes will be end filed and rewound. The entire contents of the memory and the condition of the control console will be dumped onto tape one by means of the NYU Save and Restore Functions. This is used as a restart tape for supplementary runs. The program will now pause with an HPR77777. Pressing start will cause it to loop back and read more data-alteration cards.

To make a supplementary run input, tapes one and three and output tape four are loaded. The program loads from tape one and pauses with HPR77777. Pressing start causes it to loop back and read data-alteration cards. Output is similar to that of the initial run. At the end, the present contents of storage and console will again be dumped onto tape one.

Supplementary runs can be used for two different reasons. If no new activities are to be added, updating runs can be made by simulating completed activities with data-alteration cards of duration zero; thus, no new card-to-tape run is required. Second, the effect of proposed duration changes can be tested. If the change produces an acceptable change in the output, the data-alteration card can replace the data card in the data card file. If the change is not acceptable the card can be discarded.

Manpower and Editing Program

The program is divided into two main sections:

1. manpower requirement determination; and
2. editing of critical path program output.

Output tapes four and five of the critical path program are used as input.

The purpose of the manpower computation is to determine the number of men, in each skill category, that will be required on each working day of the project. Fifteen skill categories are used. Five are assigned to Argonne National Laboratory (ANL) employees and ten to C-24 contract employees (C-24 workers constitute outside labor contracted for during construction). The categories are:

1. Optical Technician - (ANL)
2. Plastics Technician - (ANL)
3. Vacuum Technician - (ANL)
4. Mechanical Technician - (ANL)
5. Draftsman - (ANL)
6. Plastic Technician - (C-24)
7. Crane Operator - (C-24)
8. Electrical Technician - (C-24)
9. Machinist - (C-24)
10. Machinist Helper - (C-24)
11. Mechanical Technician - (C-24)
12. Pipe Fitter - (C-24)
13. Rigger - (C-24)
14. Welder - (C-24)
15. Welder Helper - (C-24)

The program assumes that each activity starts and finishes at the earliest possible time. Of course, this is not strictly true. However, an activity will usually be started near its earliest starting time unless it has been delayed to expedite a more critical activity.

Two hundred record input and output buffers are used. Data from tape number five, consisting of the earliest starting and finishing times and daily manpower requirements, are read into the input buffer. A storage location is designated for each skill category-day.

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Two hundred record input and output buffers are used. Data from tape number five, consisting of the earliest starting and finishing times and daily manpower requirements, are read into the input buffer. A storage location is designated for each skill category every

For each day that the activity is in progress the daily manpower requirement is added to the contents of the corresponding skill category-day location.

Output on tape number two consists of the manpower requirement per day per skill category, the total ANL and C-24 requirement per day, and the total man days required in each skill category. Optional output on tape number three can be used for off-line punching of output data cards. These cards can be used with an automatic curve plotter to prepare graphs of manpower versus date.

Output tape number four of the critical-path program is edited to produce two reports. The first is a list of the critical activities, namely, those with zero total slack. The second is a list of subcritical activities. A subcritical activity has been arbitrarily defined as one whose total slack is less than or equal to thirty days. Following this, output tapes four and five are scanned to produce a list of activities which must be started within the following fifty working days.

The Present Status of Network Analysis on the ZGS Project

A planning and scheduling group has been created within the Particle Accelerator Division. The group consists of two staff employees and two hourly employees. Its primary purpose is to use network analysis to coordinate and expedite construction of the ZGS.

The target date for closure of the ring magnet is June 1, 1963. Present calculations indicate that on a normal, one-shift-a-day basis this cannot be attained before the end of September 1963. Using negative-slack computations, we are in the process of determining which operations must be double shifted and/or have their resources increased to attain the June first date. When the computations are completed, a closed schedule will be published. Following the release of this schedule, regular biweekly reporting by project managers will

begin. The form illustrated in Fig. 24 will be used. In anticipation of the beginning of regular biweekly reporting, a five-hour indoctrination course was given to interested personnel during the latter part of January.

For convenient reference, our network analysis system has been named PERT. More people appear to be familiar with PERT than with CPM, so that this name more quickly communicates the basic ideas involved.

We have proposed that PERT reporting be extended to aspects of construction not under the direct control of ANL, specifically to the fabrication of components by outside contractors. A contractor's manual has been prepared which describes the basic philosophy of our system and enumerates the contractor's responsibilities and the assistance to be given to the contractor by ANL. The first application in this direction is expected to be the fabrication of the straight-section vacuum boxes. The requirement of PERT reporting is being written into the specifications of the achromatic magnet blocks. Westinghouse has volunteered to introduce PERT into the fabrication of the ring-magnet coils, a project already under way.

The computations have already disclosed a potential bottleneck. Original plans called for some preliminary work to be done on the ring-magnet coils in a workspace adjacent to octant number seven, the coil to remain in this area until it was placed in its octant. Calculation showed that this plan would introduce a delay of five months. The plan was revised to permit this work to be performed in the proton area. This bottleneck undoubtedly would have been discovered eventually, but the network analysis did uncover it and in sufficient time to permit alteration of the original plan.

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The computations have already disclosed a potential bottleneck. Original plans called for some preliminary work to be done on the ring-magnet coils in a workspace adjacent to where they were to be stored, the coils to remain in that area until it was time to place them in position. Calculation showed that this plan would introduce a delay in the schedule. The plan was revised to permit this work to be performed in the ring-magnet area. This bottleneck undoubtedly would have been discovered eventually, but the network analysis did uncover it and in sufficient time to permit alteration of the original plan.

_________ Bi-Weekly Report

_________ Monthly Report

_________ Special Report

Date: _______________

Originator: _______________

PERT STATUS REPORT

Job Number		Job Title	Job Duration Change		Fill In If Job Was Finished or Started Since Last Report		Change Of Manpower Requirement			Scheduling Office Use Only			
Tail No. (i)	Head No. (j)		From (Days)	To (Days)	Date Started	Date Finished	Category	From	To	Network Diagram	Job File Card	Job Data Card	Job Des. Card

1. Optical Technician - (ANL)
2. Plastics Technician - (ANL)
3. Vacuum Technician - (ANL)
4. Mechanical Technician - (ANL)
5. Draftsman - (ANL)
6. Plastic Technician - (C-24)
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14. Welder - (C-24)
15. Welder Helper - (C-24)

P&S 3

Fig. 24. PERT Status Report

The total cost of setting up the PERT system at ANL has been approximately forty-seven hundred dollars. In terms of the total cost of the ZGS, this amounts to roughly one hundredth of one percent of the project cost. Thirty-nine hundred dollars have been for salaries. An industrial seminar on PERT, held in January, attended by four staff members cost four hundred dollars. Time on the 704 at a rate of ninety dollars an hour has amounted to an estimated four hundred dollars.

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CHAPTER VIII
GOVERNMENT AND INDUSTRIAL USE OF PERT AND CPM

The use of techniques of network analysis is spreading through government agencies and private industry. PERT has been credited with cutting over two years from the Polaris development program. Although this figure has been disputed, the generally acknowledged success of PERT has caused it to be extended to other military research, development, and construction projects.[1] The largest of these projects is the construction of hardened, underground complexes for launching Atlas, Titan, and Minuteman intercontinental missiles. Other programs in which PERT is used are:

1. Eagle air-to-air missile, Navy;
2. Typhon antiaircraft missile, Navy;
3. Minuteman intercontinental ballistic missile, Air Force;
4. Skybolt (GAM-87A) air-to-ground ballistic missile, Air Force;
5. Nike-Zeus antimissile missile, Army.

The National Aeronautics and Space Administration has adopted a single-time-estimate form of PERT to prepare time and fiscal information on the civilian space program. The Atomic Energy Commission has specified the use of PERT in the design and construction of the ten-thousand-foot linear accelerator at Stanford University. General Electric's Light Military Electronics Department (LMED) and the Westinghouse Air Arm Division were introduced to PERT through the Polaris Program and have adopted it to many non-Polaris projects. A representative of General Electric is quoted as stating that in practically every instance where LMED applied PERT to a program

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[1] P. Geddes, "How Good Is PERT?", Aerospace Management, 4(1961) September, pp 41-43.

already under way, it discovered future problem areas in scheduling which were unknown to project managers.[1]

Private industry has in general favored the Critical Path Method. Projects on which it has been used include construction of chemical plants, office and apartment buildings, sewage treatment plants, and a Broadway play. Companies using CPM include: Allied Chemical, Anglin-Norcross, Catalytic Construction, Dow Chemical, du Pont, General Electric, Ford Motor Company, International Business Machines, Olin-Mathieson, Perini Limited, RCA, Sperry Rand, Union Carbide, and Washington Gaslight.

Some management consulting organizations are prepared to set up a PERT or CPM reporting system within a company. This service usually includes an indoctrination course for the company's employees. Five-day, in-plant training and indoctrination courses for fifteen to twenty men average around three thousand dollars. The major computer manufacturers now offer to process PERT/CPM networks on a service bureau basis. The minimum cost is in the neighborhood of seventy-five dollars.

Some large organizations with their own computing facilities have developed programs. Among these are Aerojet-General for the IBM 704, Lockheed for the IBM 709/7090, and Sperry Gyroscope for the UNIVAC II.

The cost of using PERT/CPM is low when compared with the benefits obtained. Navy experience indicates that the cost of PERT averages about 0.1% of contract price, slightly higher for small contracts.[1] This does not take into consideration possible savings due to the elimination of other reporting methods.

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[1]P. J. Klass, "PERT/PEP Management Tool Use Grows", Aviation Week, 73 (1960), November 28, pp 85-91.

CHAPTER IX
CONCLUSIONS

1. Because of the development of modern equipment for data processing, techniques of network analysis such as PERT and CPM will play an increasingly important role in the planning and scheduling of one-time engineering projects.

2. The most difficult aspect of PERT/CPM is the preparation of an accurate and representative network diagram. This requires a project manager with a thorough knowledge of his field. The networks must contain all activities which significantly constrain the end event.

3. For initial planning and resource management, the activity-oriented diagram is superior to the event-oriented diagram.

4. Diagrams should be numbered randomly or sequentially with some numbers missing. The author's preference is the sequential-missing number system because of its inherent logic and ease of data processing.

5. The merits of three time estimates versus one have not yet been resolved. The principal effect of using three time estimates is to cause a more pessimistic prediction. Recent trends appear to be toward the use of one estimate.

6. Some error can be tolerated in the initial estimates of activity duration. Early calculations will reveal which areas are critical or subcritical. These areas may then be examined more closely to improve the accuracy of the original estimates.

7. Network diagramming is not suitable to open-end, production-type projects for which more traditional methods, such as line-of-balance, can be used.

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7. Network diagramming is not suitable to open-end, production-type projects for which more traditional methods, such as line-of-balance, can be used.

8. The use of PERT/CPM need not be costly. Military experience indicated that a cost of 0.1% of contract price is representative.

9. PERT/CPM has uncovered unknown problem areas in almost every situation in which it has been applied.

APPENDICIES

APPENDIX A
VARIABLES USED IN FORTRAN SOURCE PROGRAMS

<u>Dimensioned</u>

I	Activity tail number
IA	1. Tail number of activity description
	2. Event or milestone number
IDATE	Calendar date
IO	Tail number of activity in output buffer
J	Activity head number
JA	Head number of activity description
JD	Activity duration (days)
JDATE(1)	Earliest starting date
JDATE(2)	Latest starting date
JDATE(3)	Earliest finishing date
JDATE(4)	Latest finishing date
JDATEO	Value of JDATE in output buffer
JDES	Alpha-numeric activity description
JDESO	Value of JDES in output buffer
JD0	Value of JD in output buffer
JEFT	Earliest finishing time (days)
JEST	Earliest starting time (days)
JESTO	Value of JEST in output buffer
JKMIN	Indexing parameter used to determine starting point of comparison loop during backward pass
JLFT	Latest finishing time
JO	Value of J in output buffer
JSCHED	Scheduled date of project milestone
MP	Manpower required per activity
MPANL	ANL manpower required per day
MPCAT	Total manpower required per skill category
MPC24	C-24 Contract manpower required per day

MPO	Value of MP in output buffer
MPR	Total manpower required per skill category per day
MTFT	Total slack
MTFTO	Value of MTFT in output buffer

Nondimensioned

IAOLD	Previous value of IA
INEW	1. Value of I on data-alteration card 2. Number of day since last run used to update file of calendar dates
IOLD	Previous value of I
I1	Index of record being read from tape 2 input buffer
I2	Index of record being read from tape 3 input buffer
I3	Number of record to be read into tape 2 input buffer
I4	Total number of MP records from tape 2 read
I5	Total number of JDES records from tape 3 read
I6	Number of records to be read into tape 3 input buffer
I7	Number of record currently in output buffer
I8	Number of milestone records in input buffer
I9	Number of milestone records in output buffer
I10	Total number of milestone records read
I11	Total number of records on tape 5
I12	Number of records to be read into input buffer - manpower program
I13	Total number of records read into input buffer - manpower program
JAOLD	Previous value of JA
JDNEW	New value of JD on data-alteration card
JLST	Latest starting time
JNEW	Value of J on data-alteration card
JOLD	Previous value of J

K1 Activity index at which backward pass is started

LJLFT Arbitrary time (days) of activity completion on data-alteration card

MFLOAT Maximum value of total float used for editing tape 4

MJEFT Maximum value of JEFT

MLFT Maximum value of JLFT

ND Number of calendar date records on tape 2

NET Node early time

NJ Number of activity-data records on tape 2

NJD Number of activity-description records on tape 3

NLT Node late time

NM Number of milestone records on tape 3

NQ Control digit punched in column 70 of data-alteration card:

1. Read new value of JD; do not make forward or backward pass
2. Read new value of JD, and make a forward and backward pass
3. Read in arbitrary completion time and make backward pass; no output
4. Read in arbitrary completion time and make backward pass; output
5. End of run; rewind tapes and stop
6. Update calendar dates

N1 Indexing parameter used in milestone computation

Y Dummy variable used to call SOS function

APPENDIX B
FORTRAN LISTING OF DATA-CHECKING PROGRAM

```
C       JOHN C. POLLOCK, 1246/PAD-133
C       DATA CHECKING PROGRAM
C       SENSE SWITCH SETTINGS
C       1-DOWN, DOES NOT CHECK TAPE 3 FOR JOB DESCRIPTION
C       ERRORS
C       2-DOWN, DOES NOT CHECK TAPE 3 FOR MILESTONE CARD
C       ERRORS
C       TAPES USED
C       TWO-INPUT
C       THREE-INPUT
        DIMENSIONI(4700), J(4700)
        IOLD=0
        JOLD=0
  4     FORMAT(4I5)
        READ4, NJ, NJD, ND, NM
  5     FORMAT(2I5)
        READINPUTTAPE2, 5, (I(K), J(K), K=1, NJ)
  7     FORMAT
        RESTORE
            JOHN C. POLLOCK, BLDG. 360
        SPACE
            ON LINE PRINT OF ERRONEOUS DATA OF 1246/PAD 133
        SPACE
            IOLD          JOLD          I          J          ERROR
        SPACE
        ENDOFFORMAT
        PRINT 7
 12     FORMAT
        SPACE 1
```

```
          -I          -I                               FIRST CARD
        SPACE 1
        ENDOFFORMAT
        K1=NJ
18      PRINT12, I(1), J(1)
        DO32K=1, K1
        IF(J(K)-I(K) )20, 20, 22
 8      FORMAT
          -I          -I          -I             -I TAIL NUMBER EXCEEDS
       X HEAD NUMBER
        ENDOFFORMAT
20      PRINT8, IOLD, JOLD, I(K), J(K)
        GOTO30
22      IF(I(K)-IOLD) 26, 24, 30
24      IF(J(K)-JOLD)26, 28, 30
 9      FORMAT
          -I          -I          -I             -I SEQUENCE ERROR
        ENDOFFORMAT
26      PRINT9, IOLD, JOLD, I(K), J(K)
        GOTO30
10      FORMAT
          -I          -I          -I             -I DUPLICATE CARD
        ENDOFFORMAT
28      PRINT 10, IOLD, JOLD, I(K), J(K)
30      IOLD=I(K)
32      JOLD=J(K)
        REWIND2
33      FORMAT
        SPACE1
          -I            -I                       LAST CARD
```

```
      SPACE
      ENDOFFORMAT
      PRINT33, I(K1), J(K1)
      IF(SENSESWITCH1) 140, 37
37    IF(SENSELIGHT 1) 42, 38
38    DO39K=1, NJ
      I(K)=0
39    J(K)=0
      IOLD=0
      JOLD=0
      READINPUTTAPE3, 5, (I(K), J(K), K=1, NJD)
      K1=NJD
      SENSELIGHT1
40    FORMAT
      SPACE3
            JOB DESCRIPTION CARD ERRORS
      SPACE2
      ENDOFFORMAT
      PRINT40
      GOTO18
42    IF(SENSESWITCH2)60, 44
44    DO45K=1, NJ
45    I(K)=0
46    FORMAT
      SPACE3
            MILESTONE CARD ERRORS
      SPACE3
            IOLD         I(K)
      ENDOFFORMAT
      PRINT46
 3    FORMAT(I5)
47    READINPUTTAPE3, 3, (I(K), K=1, NM)
```

```
        IOLD=0
        DO56K=1,NM
        IF(I(K)-IOLD)54,50,56
 48     FORMAT
              -I           -I                DUPLICATE CARD
        ENDOFFORMAT
 50     PRINT48,IOLD,I(K)
        GOTO56
 52     FORMAT
              -I           -I                INCORRECT SEQUENCE
        ENDOFFORMAT
 54     PRINT52,IOLD,I(K)
 56     CONTINUE
 58     FORMAT
        SPACE 1
              -I                             LAST MILESTONE CARD
        ENDOFFORMAT
        PRINT58,I(NM)
 60     REWIND3
140     STOP77777
        END(0,1,0,0,1)
```

APPENDIX C
FORTRAN LISTING OF CRITICAL PATH PROGRAM

```
C        CRITICAL JOB DETERMINATION
C        TAPES USED
C        ONE-INPUT-OUTPUT, SOSF TAPE
C        TWO-INPUT (JOB DATA, DATE DATA)
C        THREE-INPUT (JOB DESCRIPTIONS)
C        FOUR-OUTPUT (FOR OFF-LINE PRINTING)
C        FIVE OUTPUT, DATA FOR MANPOWER COMPUTATION
C        SENSE SWITCH SETTINGS
C        ONE-DOWN, DOE NOT REWIND TAPE 4
C        TWO, UP MANPOWER OUTPUT ON TAPE 5
C        TWO, DOWN NO MANPOWER OUTPUT ON TAPE 5
C        THREE, DOWN-READ CARDS DURING INITIAL RUN
C        FOUR, UP-MILESTONE PRINTOUT
C        FOUR, DOWN-NO MILESTONE PRINTOUT
C        FIVE, UP-JOB DATA CARD ERRORS PRINTED
C       XON LINE
         DIMENSIONI(4500), J(4500), JD(4500),
        XJKMIN(4500), JEST(4500), JLFT(4500),
        XIDATE(650), IO(10), JO(10), JDO(10),
        XJDES(10, 10), JDESO(10, 10), JEFT(10), JESTO(10),
        XMTFT(10), JDATE(4, 10), MP(15, 10), MPO(15, 10),
        XJSCHED(10), IA(10), JA(10)
  1500   FORMAT (415)
         READ1500, NJ, NJD, ND, NM
C        NJ=TOTAL NUMBER OF JOB DATA CARDS
C        NJD=TOTAL NUMBER OF JOB DESCRIPTION CARDS
C        ND=TOTAL NUMBER OF CALENDAR DATE CARDS
C        NM=TOTAL NUMBER OF MILESTONE CARDS
C        INITIALIZE STORAGE
         NQ=5
```

```
19      DO20K=1, NJ
        JEST(K)=0
20      JKMIN(K)=0
        IOLD=0
        JOLD=0
C       DATA INPUT
 5      FORMAT (2I5, I4)
        READINPUTTAPE2, 5, (I(K), J(K), JD(K),
       XK=1, NJ)
 6      FORMAT (1X, A6)
        READINPUTTAPE2, 6, (IDATE(K), K=1, ND)
        REWIND2
C       CHECK FOR SEQUENCE AND DUPLICATE DATA
 7      FORMAT
        RESTORE
            JOHN C. POLLOCK, BLDG. 360, X2542
        SPACE
            ON LINE PRINT OF ERRONEOUS DATA OF 1246/PAD 133
        SPACE
            IOLD      JOLD       I(K)       J(K)        K
        SPACE
        ENDOFFORMAT
        PRINT7
48      DO80K=1, NJ
        IF(J(K)-I(K))60, 60, 49
49      IF(I(K)-IOLD)60, 50, 70
50      IF(J(K)-JOLD)60, 60, 70
60      SENSELIGHT1
 8      FORMAT(I13, 4I9)
        PRINT8, IOLD, JOLD, I(K), J(K), K
70      IOLD=I(K)
```

```
   80     JOLD=J(K)
          IF(SENSELIGHT1)90,91
   90     STOP22222
C         COMPUTE MIN VAL OF SUBSCRIPT K FOR
C        XIDENTICAL J
   91     DO130K=1,NJ
   92     IF(JKMIN(K))130,100,130
  100     JKMIN(K)=K
  101     DO120L=K,NJ
          IF(J(K)-I(L))130,130,102
  102     IF(J(K)-J(L))120,110,120
  110     JKMIN(L)=K
  120     CONTINUE
  130     CONTINUE
          SENSELIGHT 1
          IF(SENSESWITCH3) 140,220
C         INSERT NEW DATA INTO MEMORY
C         NQ=1, READ NEW VALUE OF JD(K).  DO NOT MAKE
C         FORWARD OR BACKWARD PASS
C         NQ=2, READ NEW VALUE OF JD(K), MAKE FORWARD
C          AND BACKWARD PASS
C         NQ=3, READ IN ARBITRARY JLFT(K).  MAKE BACKWARD
C         PASS.  NO PRINT OUT
C         NQ=4, READ IN ARBITRARY JLFT(K).  MAKE BACKWARD
C         PASS.  PRINT OUT.
C         NQ=5, END OF RUN.  REWIND TAPES AND STOP
C         NQ=6, UPDATE CALENDAR DATES
   12     FORMAT(2I5, I4, 3X, I3, 49X, I1)
  140     READ12, INEW, JNEW, JDNEW, LJLFT, NQ
          GOTO(161, 141, 161, 161, 1252, 1402), NQ
 1402     ND=ND-INEW
```

```
      DO1403K=1,ND
      L=K+INEW
 1403 IDATE(K)=IDATE(L)
      GOTO140
  141 DO150K=1,NJ
      JEST(K)=0
  150 JLFT(K)=0
  161 DO190K=1,NJ
  162 IF(INEW-I(K))190, 170, 190
  170 IF(JNEW-J(K))190, 180, 190
  180 GOTO(182, 182, 200, 200), NQ
  182 JD(K)=JDNEW
      GOTO(140, 220, 140, 140), NQ
  190 CONTINUE
  200 IF(LJLFT-JLFT(K))202, 140, 140
  202 JLFT(K)=LJLFT
      K1=K
  204 GOTO310
C     COMPUTE EARLIEST STARTING TIMES
  220 DO260K=1,NJ
      NET=JEST(K)+JD(K)
  221 DO250L=K,NJ
      IF(I(L)-J(K))250, 230, 260
  230 IF(JEST(L)-NET)240, 260, 260
  240 JEST(L)=NET
  250 CONTINUE
  260 CONTINUE
C     COMPUTE MAX VALUE OF JEFT
      MLFT=JEST(1)+JD(1)
  261 DO290K=2,NJ
      JLFT(K)=JEST(K)+JD(K)
```

```
      IF(JLFT(K)-MLFT)290,290,280
 280  MLFT=JLFT(K)
 290  CONTINUE
C     SET ALL JLFT (K) EQUAL TO MLFT
 291  DO300K=1,NJ
 300  JLFT(K)=MLFT
C     COMPUTE LATEST FINISHING TIMES
 309  K1=NJ
 310  DO370K=1,K1
      L=K1+1-K
      NLT=JLFT(L)-JD(L)
 311  DO330M=1,L
      N=L+1-M
      IF(J(N)-I(L))330,320,330
 320  IF(NLT-JLFT(N))340,370,370
 330  CONTINUE
      GOTO370
 340  M=JKMIN(N)
 341  DO360I=M,N
      IF(J(I)-J(N))360,350,360
 350  JLFT(I)=NLT
 360  CONTINUE
 370  CONTINUE
      GOTO(384,384,140,384,384,384),NQ
C     OUTPUT VALUES FOR ALL JOBS
C     INDEX I1=VALUE OF MP(M,I1) BEING READ FROM INPUT
C     BUFFER
C     INDEX I2=SUBSCRIPT OF JDES(M,I2) BEING READ FROM
C     INPUT BUFFER
C     INDEX I3,=NUMBER OF MP(L,I) RECORDS TOBEREAD INTO
C     INPUT BUFFER
```

```
C        INDEX I4= TOTAL NUMBER OF MP(L,I) RECORDS READ
C        TO DATE
C        INDEX I5= TOTAL NUMBER OF JDES(I,L) RECORDS READ
C        TO DATE
C        INDEX I6= NUMBER OF JDES(I,L) RECORDS TO BE READ
C        INTO INPUT BUFFER
C        INDEX I7= SUBSCRIPT OF OUTPUT DATA IN OUTPUT
C        BUFFER
C        INDEX I8= NUMBER OF MILESTONE RECORDS IN INPUT
C        BUFFER
C        INDEX I9= NUMBER OF MILESTONE RECORDS IN OUTPUT
C        BUFFER
C        INDEX I10= TOTAL NUMBER OF MILESTONE RECORDS
C        READ
C        INDEX I10= TOTAL NUMBER OF RECORDS ON TAPE 7
 384     IAOLD=0
         JAOLD=0
 10      FORMAT
         RESTORE
     THIS OUTPUT IS FOR
                 JOHN C. POLLOCK, BLDG. 360, X2452, 1246/PAD 133
         SPACE3
XINATION                    ZGS ASSEMBLY, CRITICAL PATHDETERM
         SPACE 2
    JOB NUMBER           JOB DESCRIPTION                DU
XRATION        STARTING DATE           FINISH DATE  SLACK
     I      J
X(DAYS)          EARLY    LATE      EARLY      LATE    (DAYS)
         SPACE
         ENDOFFORMAT
         WRITEOUTPUTTAPE4, 10
         IF(SENSESWITCH5) 386, 385
```

```
17     FORMAT
                                          ERRORS IN JOB DESCRIPTION FILE
       SPACE
           IOLD          JOLD          I(K)          J(K)
       ENDOFFORMAT
385    PRINT17
386    I1=1
       I2=1
       I3=10
       I4=0
       I5=0
387    I6=10
       I7=1
       I11=0
388    SENSELIGHT2
       SENSELIGHT3
       DO 1084 K=1,NJ
       IF(SENSELIGHT2) 1002, 1004
 701   FORMAT (2I5, 1X, 9A6, A1)
1002   READINPUTTAPE3, 701, (IA(L), JA(L),
      X(JDES(I, L), I=1, 10), L=1, I6)
1004   IF(SENSELIGHT3)1006, 1010
1006   IF(SENSESWITCH2)1010, 1008
   9   FORMAT(14X, 15I3)
1008   READINPUTTAPE2, 9, (MP(L, I), L=1, 15),
      XI=1, I3)
1010   IF(JD(K))1012, 1078, 1012
1012   I11=I11+1
       DO1014M=1, 15
1014   MPO(M, I7)=MP(M, I1)
1016   IF(JA(I2)-IA(I2))1022, 1022, 1018
```

```
1018  IF(IA(I2)-IAOLD)1022, 1020, 1026
1020  IF(JA(I2)-JAOLD)1022, 1024, 1026
1022  IF(SENSESWITCH 5)1026, 1023
  15  FORMAT
            -I       -I       -I          -I INCORRECT SEQUENCE
      ENDOFFORMAT
1023  PRINT 15, IAOLD, JAOLD, JAOLD, IA(I2), JA(I2)
      GOTO1026
1024  SENSELIGHT2
      IF(SENSESWITCH5) 1026, 1025
  16  FORMAT
            -I       -I       -I          -I DUPLICATE CARD
      END OF FORMAT
1025  PRINT 16, IAOLD, JAOLD, IA(I2), JA(I2)
1026  IF(I(K) - IA(I2))1040, 1028, 1029
1028  IF(J(K)-JA(I2))1040, 1042, 1029
1029  IF(SENSELIGHT2) 1033, 1030
1030  IF(SENSESWITCH5)1032, 1031
  18  FORMAT
                              -I          -I NO CORRESPONDING
      DATA CARD
      ENDOFFORMAT
1031  PRINT 18, IA(I2), JA(I2)
1032  IAOLD=IA(I2)
      JAOLD=JA(I2)
1033  I5=I5+1
      IF(I6-I2) 1034, 1035, 1034
1034  I2=I2+1
      GOTO1016
1035  I2=1
      IF((NJD-I5)-10) 1036, 1002, 1002
1036  I6=NJD-I5
```

```
1038  GOTO1002
1040  SENSELIGHT4
1042  JEFT(I7)=JEST(K)+JD(K)
      JLST=JLFT(K)-JD(K)
      MTFT(I7)=JLST-JEST(K)
      IF(JEST(K))1051, 1050, 1051
1050  JDATE(1, I7)=IDATE(1)
      GOTO1049
1051  L=JEST(K)
      JDATE(1, I7)=IDATE(L)
1049  IF(JLST)1052, 1052, 1053
1052  JDATE(2, I7)=IDATE(1)
      GOTO1054
1053  JDATE(2, I7)=IDATE(JLST)
1054  L=JEFT(I7)
      JDATE(3, I7)=IDATE(L)
      IF(JLFT(K))1055, 1055, 1056
1055  JDATE(4,I7)=IDATE(1)
      GOTO1057
1056  L=JLFT(K)
      JDATE(4, I7)=IDATE(L)
1057  I0(I7)=I(K)
      J0(I7)=J(K)
      JDO(I7)=JD(K)
      IF(SENSELIGHT4)1058, 1060
1058  DO1059N=1, 10
1059  JDESO(N, I7)=0
      SENSELIGHT4
      GOTO1064
1060  DO1062N=1, 10
1062  JDESO(N, I7)=JDES(N, I2)
```

```
1064   JESTO(17)=JEST(K)
       IF(17-10)1070,1066,1070
1066   I7=0
  11   FORMAT (2I6, 2X, 9A6, A1, I6, A11, A8, A9, A8, AI7)
       WRITEOUTPUTTAPE4, 11, (IO(N),JO(N),
      X(JDESO(M, N), M=1, 10), JDO(N), (JDATE
      X(M, N), M=1,4), MTFT(N), N=1, 10)
       IF(SENSESWITCH2) 1070, 1068
   3   FORMAT (17I3)
1068   WRITEOUTPUTTAPE5, 3, (JESTO(N), JEFT(N),
      X(MPO(M, N), M=1, 15), N=1, 10)
1070   I7=I7+1
       IF(SENSELIGHT4) 1078, 1071
1071   IAOLD=IA(I2)
       JAOLD=JA(I2)
       I5=I5+1
       IF(I6-I2)1075, 1072, 1075
1072   I2=1
       IF((NJD-I5 ) -10) 1073, 1074, 1074
1073   I6=NJD-I5
1074   SENSELIGHT2
       GOTO1078
1075   I2=I2+1
1078   I1=I1+1
       I4=I4+1
       IF(11-I1)1084, 1080, 1084
1080   I1=1
       SENSELIGHT3
       IF((NJ-I4 )-10)1082, 1084, 1084
1082   I3=NJ-I4
1084   CONTINUE
```

```
      I7=I7-1
      IF(I7)1200,1200,1086
 1086 WRITEOUTPUTTAPE4,11,(IO(N),JO(N),
     X(JDESO(M,N),M=1, 10),JDO(N),(JDATE(M,N),
     XM=1,4),MTFT(N),N=1, I7)
      IF(SENSESWITCH2)1200,1088
 1088 WRITEOUTPUTTAPE5, 3, (JESTO(N), JEFT(N),
     X(MPO(M,N),M=1, 15),N=1,I7)
 1200 IF(SENSESWITCH4) 1250,1202
 1201 FORMAT
      RESTORE
                ZGS ASSEMBLY, MAJOR MILESTONES
      SPACE3
        EVENT             EVENT DESCRIPTION
X      EARLIEST SCHEDULED LATEST EVENT SLACK
        NUMBER
X        DATE         DATE       DATE      (DAYS)
      SPACE2
      ENDOFFORMAT
 1202 WRITEOUTPUTTAPE4,1201
      IF(NM-10) 1204, 1206,1206
 1204 I8=NM
      GOTO1207
 1206 I8=10
 1207 N1=1
      I9=1
      I10=1
      SENSELIGHT2
 1208 DO1244K=1,NM
      IF(SENSELIGHT2) 1211, 1212
 1210 FORMAT (I5, 6X,9A6,A1, 3X, A6)
```

```
1211   READINPUTTAPE3, 1210, (IA(J), (JDES(L, J),
      XL=1, 10), JSCHED(J), J=1, I8)
1212   DO1216N=N1, NJ
       IF(IA(I9)-I(N))1216, 1214, 1216
1214   I1=JEST(N)
       JDATE(1, I9)=IDATE(I1)
       N1=N
       GOTO1218
1216   CONTINUE
1218   DO1230N=1, NJ
       M=N1-N
       IF(IA(I9)-J(M))1230, 1220, 1230
1220   I2=JLFT(M)
       IF(I2) 1222, 1222, 1226
1222   JDATE(2, I9)=IDATE(1)
       GOTO 1228
1226   JDATE(2, I9)=IDATE(I2)
1228   MTFT(I9)=I2-I1
       GOTO1231
1230   CONTINUE
1231   IF(I9-I8)1240, 1233, 1240
1233   IF((NM-I10)-10) 1234, 1235, 1235
1234   I8=NM-I10
1235   SENSELIGHT2
1236   FORMAT(I14, 5X, 9A6, A1, A12, A11, A9, I9)
       WRITEOUTPUTTAPE4, 1236, (IA(J), (JDES
      X(L, J), L=1, 10), JDATE(1, J), JSCHED(J),
      XJDATE(2, J), MTFT(J), J=1, I9)
       I9=1
       GOTO1242
1240   I9=I9+1
```

```
1242   I10=I10+1
1244   CONTINUE
1250   REWIND3
       IF(SENSELIGHT 1) 1252, 1251
1251   GOTO(140, 140, 140, 140, 1252), NQ
1252   IF(SENSESWITCH2) 1254, 1253
1253   WRITEOUTPUTTAPE 5, 6, (IDATE(K), K=1, ND)
       REWIND5
       REWIND2
       PUNCH5, I 11, ND
1254   IF(SENSESWITCH1) 1270, 1262
1262   ENDFILE4
       REWIND4
       Y=SOSF(1)
1270   PAUSE77777
       GOTO140
       END(0, 1, 0, 0, 1)
```

APPENDIX D
FORTRAN LISTING OF MANPOWER DETERMINATION AND OUTPUT EDITING PROGRAM

```
C        TAPES USED
C        TWO-OUTPUT
C        THREE-OUTPUT FOR OFF-LINEPUNCHING
C        FOUR-INPUT FOR EDITING
C        FIVE-INPUT FOR MANPOWER COMPUTATION
C        SENSESWITCH SETTINGS
C        ONE-DOWN, NO OUTPUT ON TAPE5
C        TWO-DOWN, DOES NOT EDIT FOR
C        ACTIVITIES WHICH MUST START IN NEXT
C        FIFTY DAYS
C        THREE-DOWN, DOES NOT EDIT TAPE4
         DIMENSIONIDATE(750), MPR (15,750),MP
       X(15,200),JEST(200),JEFT(200),MPANL(750),
       XMPC24(750), MPCAT(15),I(200),J(200),
       XJD(200),JDES(10,200),JDATE(4,200), MTFT (200),
       XJLST(200),IO(200),JO(200),JDO(200),
       XJDESO(10,200),JDATEO(4,200),MTFTO(200)
  3      FORMAT(2I5)
         READ3, I11, ND
C        ZERO OUT STORAGE
 20      DO22K=1,I11
         DO22L=1,15
 22      MPR(L,K)=0
         MJEFT=0
         IF(I11-200)25,26,26
 25      I12=I11
         GOTO27
 26      I12=200
```

```
27      I13=0
        I14=0
 6      FORMAT(17I3)
29      READINPUTTAPE5,6 (JEST(K), JEFT(K),
       X(MP(J,K), J=1, 15), K-1,I12)
        I13=I13+I12
        DO30J=1,I12
        MJEFT=XMAX0F(MJEFT, JEFT(J))
30      JEFT(J)=JEFT(J)-1
        DO32J=1,I12
        I15=JEST(J)
        I16=JEFT(J)
        DO32I=I15,I16
        DO32L=1,15
32      MPR(L,I)=MPR(L,I)+MP(L,J)
        IF((I11-I13)-200)33,29,29
33      IF(I11-I13)36,36,34
34      I12=I11-I13
        GOTO29
36      DO42I=1,MJEFT
        DO40J=1,5
40      MPANL(I)=MPANL(I)+MPR(J,I)
        DO42J=6,15
42      MPC24(I)=MPC24(I)+MPR(J,I)
        DO44I=1,15
        DO44J=1,MJEFT
44      MPCAT(I)=MPCAT(I)+MPR(I,J)
 5      FORMAT(1X, A6)
        READINPUTTAPE5,5(IDATE(K), K=1,ND)
        REWIND5
 4      FORMAT
```

```
      RESTORE
      THIS OUTPUT IS FOR
                     JOHN C. POLLOCK, BLDG. 360 1246/PAD 133
      SPACE3
                            ZGS ASSEMBLY, CRITICAL PATH CAL
     XCULATION OF MANPOWER REQUIREMENTS
      SPACE2
          DATE        MANPOWER REQUIRED
      SPACE2
          CATEGORY  1  2  3  4  5  6  7  8  9
X    10  11  12  13  14  15  DAILY TOTAL
X                   ANL  C-24
      SPACE
      ENDOFFORMAT
      WRITEOUTPUTTAPE2,4
  7   FORMAT(8X,A6,I18,14I5,I8,I6)
      WRITEOUTPUTTAPE2,7,(IDATE(K),(MPR
     X(J,K), J=1,15), MPANL(K), MPC24(K), K=1, MJEFT)
  8   FORMAT
      SPACE3
                            TOTAL MAN-DAYS PER SKILL CATEG
     XORY
      SPACE1
                        CATEGORY 1, OPTICAL TECHNICIAN(ANL)
X      -I MAN-DAYS
                       CATEGORY 2, PLASTICS TECHNICIAN (ANL)
X      -I MAN-DAYS
                        CATEGORY 3, VACUUM TECHNICIAN (ANL)
X      -I MAN-DAYS
                    CATEGORY 4, MECHANICAL TECHNICIAN (ANL)
X      -I MAN-DAYS
```

```
                 CATEGORY 5, DRAFTSMAN (ANL)
X       -I MAN-DAYS
                 CATEGORY 6, PLASTICS TECHNICIAN (C-24)
X       -I MAN-DAYS
                 CATEGORY 7, CRANE OPERATOR (C-24)
X       -I MAN-DAYS
                 CATEGORY 8, ELECTRICAL TECHNICIAN (C-24)
X       -I MAN-DAYS
                 CATEGORY 9, MACHINIST (C-24)
X       -I MAN-DAYS
                 CATEGORY 10, MACHINIST HELPER (C-24)
X       -I MAN-DAYS
                 CATEGORY 11, MECHANICAL TECHNICIAN (C-24)
X       -I MAN-DAYS
                 CATEGORY 12, PIPE FITTER (C-24)
X       -I MAN-DAYS
                 CATEGORY 13, RIGGER (C-24)
X       -I MAN-DAYS
                 CATEGORY 14, WELDER (C-24)
X       -I MAN-DAYS
                 CATEGORY 15, WELDER HELPER (C-24)
X       -I MAN-DAYS
        ENDOFFORMAT
        WRITEOUTPUTTAPE2,8,(MPCAT(K),K=1, 15)
        IF(SENSESWITCH1)70,46
  9     FORMAT(18I4)
 46     WRITEOUTPUTTAPE3,9,((MPR(J,K),J=1,15),
       XMPANL(K),MPC24(K),K=1,MJEFT)
        ENDFILE3
        REWIND3
 70     IF(SENSESWITCH3) 154, 73
```

```
 72     FORMAT
        RESTORE
  THIS OUTPUT IS FOR
                       JOHN C. POLLOCK, BLDG. 360, 1246/PAD 133
        SPACE3
                                ZGS PROJECT, CRITICAL ACTIVIT
XIES
        SPACE2
  JOB NUMBER      JOB DESCRIPTION          DU
XRATION    STARTING DATE          FINISH DATE          SLACK
  I    J
XDAYS)     EARLY    LATE          EARLY  LATE          (DAYS)
        SPACE
        ENDOFFORMAT
 73     WRITEOUTPUTTAPE2, 72
        MFLOAT=0
        SENSELIGHT1
 75     IF(I11-200)76, 78, 78
 76     I12=I11
        GOTO80
 78     I12=200
 80     I13=0
        I15=1
200     FORMAT
        RESTORE
  DUMMY
  DUMMY
        SPACE3
  DUMMY
        SPACE2
  DUMMY
```

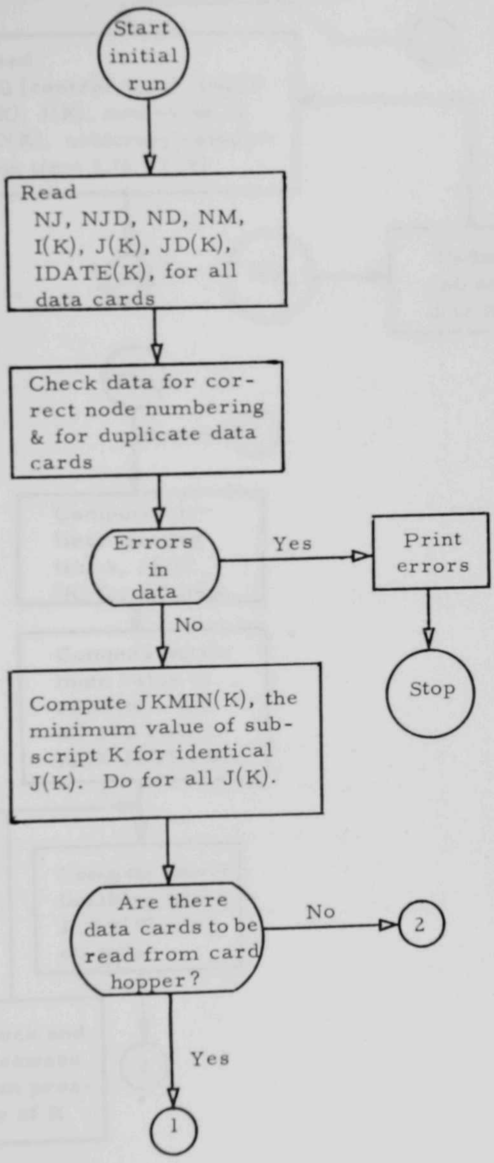
```
 DUMMY
        SPACE
        ENDOFFORMAT
        READINPUTTAPE4,200
 74     FORMAT(2I6, 2X, 9A6, A1, I6, A11, A8, A9, A8, I7)
 82     READINPUTTAPE4, 74, (I(K), J(K), (JDES(I,K),
      XI=1, 10), JD(K), (JDATE(I,K), I=1, 4),
      XMTFT(K), K=1, I12)
        I13=I13+I12
        DO102K=1, I12
        IF(MTFT(K)-MFLOAT)84, 84, 102
 84     IO(I15)=I(K)
        JO(I15)=J(K)
        JDO(I15)=JD(K)
        DO88J=1, 10
 88     JDESO(J, I15)=JDES(J,K)
        DO90J=1, 4
 90     JDATEO(J, I15)=JDATE(J,K)
        MTFTO(I15)=MTFT(K)
        IF(I15-200) 100, 96, 96
 96     WRITEOUTPUTTAPE2, 74, (I0(L), JO(L),
      X(JDESO(M,L), M=1, 10), JDO(L), (JDATEO(M,L),
      XM=1, 4), MTFTO(L), L=1,200)
        I15=1
        GOTO102
 100    I15=I15+1
 102    CONTINUE
        IF(I11-I13) 109, 109, 106
 106    IF((I11-I13)-200)108,82,82
 108    I12=I11-I13
        GOTO82
```

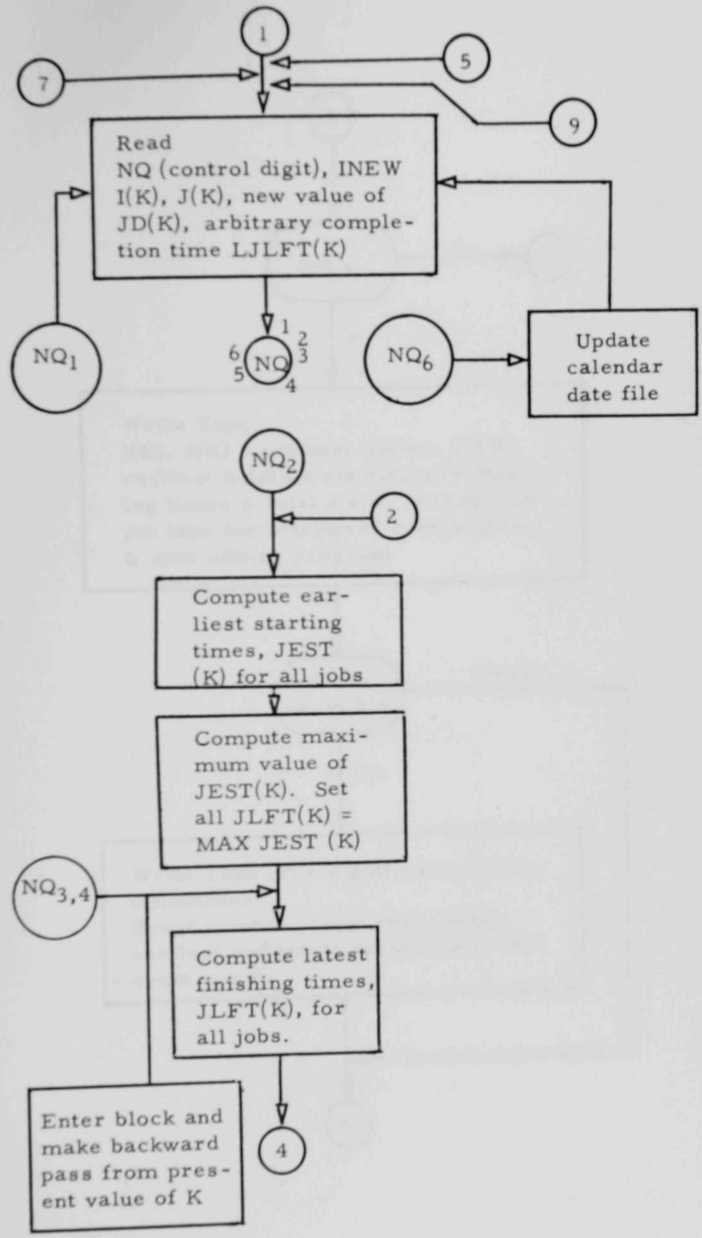
```
109     I15=I15-1
        REWIND4
        WRITEOUTPUTTAPE2, 74, (IO(L), JD(L),
       X(JDESO(M, L), M=1, 10), JDO(L), (JDATEO(M, L),
       XM=1, 4), MTFTO(L), L=1, I15)
        IF(SENSELIGHT1) 110, 118
110     MFLOAT=30
112     FORMAT
        RESTORE
  THIS OUT IS FOR
                        JOHN C. POLLOCK, BLDG. 360, 1246/PAD 133
        SPACE3
                     ZGS PROJECT, SUBCRITICAL ACTIVITIES (SLACK
XLESS THAN OR EQUAL TO THIRTY DAYS)
        SPACE2
  JOB NUMBER          JOB DESCRIPTION              DU
XRATION   STARTING DATE    FINISH DATE    SLACK
   I    J
X(DAYS)   EARLY       LATE         EARLY        LATE    (DAYS)
        SPACE
        ENDOFFORMAT
        WRITEOUTPUTTAPE2, 112
        GOTO75
116     FORMAT
        RESTORE
  THIS OUTPUT IS FOR
                        JOHN C. POLLOCK, BLDG. 360, 1246/PAD 133
        SPACE3
                     ZGS PROJECT, LIST OF ACTIVITIES WHICH MUST
XBE STARTED WITHIN THE NEXT FIFTY DAYS
        SPACE2
```

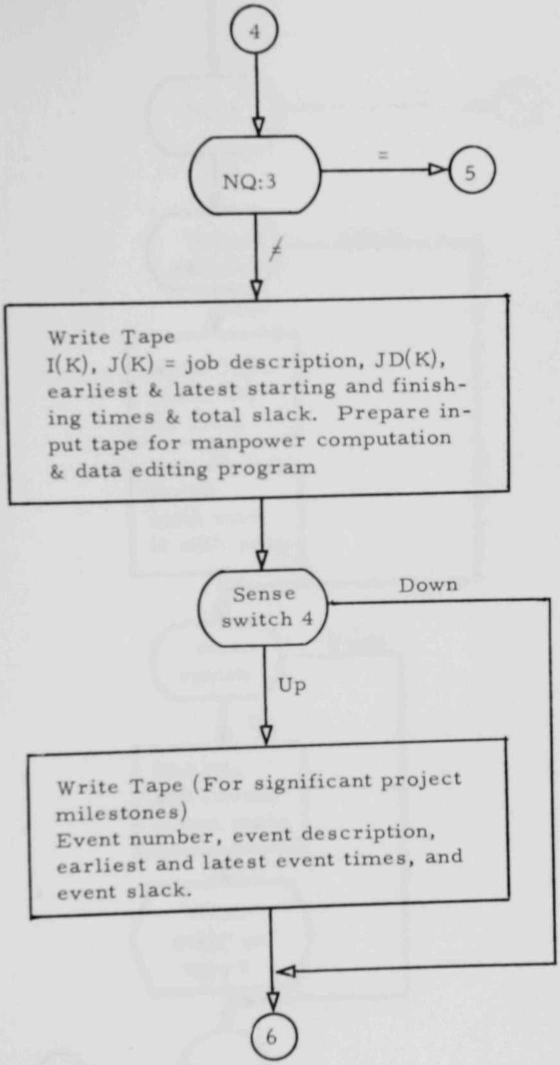
```
 JOB NUMBER           JOB DESCRIPTION            DU
XRATION    STARTING DATE      FINISH DATE      SLACK
    I    J
X(DAYS)     EARLY     LATE      EARLY      LATE    (DAYS)
        SPACE
        ENDOFFORMAT
 118    IF(SENSESWITCH2)154, 119
 119    WRITEOUTPUTTAPE2, 116
        IF(I11-200) 120, 122, 122
 120    I12=I11
        GOTO124
 122    I12=200
 124    I13=0
        I15=1
        READINPUTTAPE4, 200
 126    READINPUTTAPE4, 74, (I(K), J(K), JD, (K),
       X(JDES(I, K), I=1, 10), (JDATE(I,K), I=1,4),
       XMTFT(K), K=1, I12)
 127    FORMAT (I3)
        READINPUTTAPE5, 127, (JEST(K), K=1, I12)
        I13=I13+I12
        DO130K=1, I12
 130    JLST(K)=JEFT(K)+MTFT(K)
        DO146K=1, I12
        IF(JLST(K)-50)132, 132, 146
 132    IO(I15)=I(K)
        JO(I15)=J(K)
        JDO(I15)=JD(K)
        DO134J=1, 10
 134    JDESO(J, I15)=JDES(J,K)
        DO136J=1,4
```

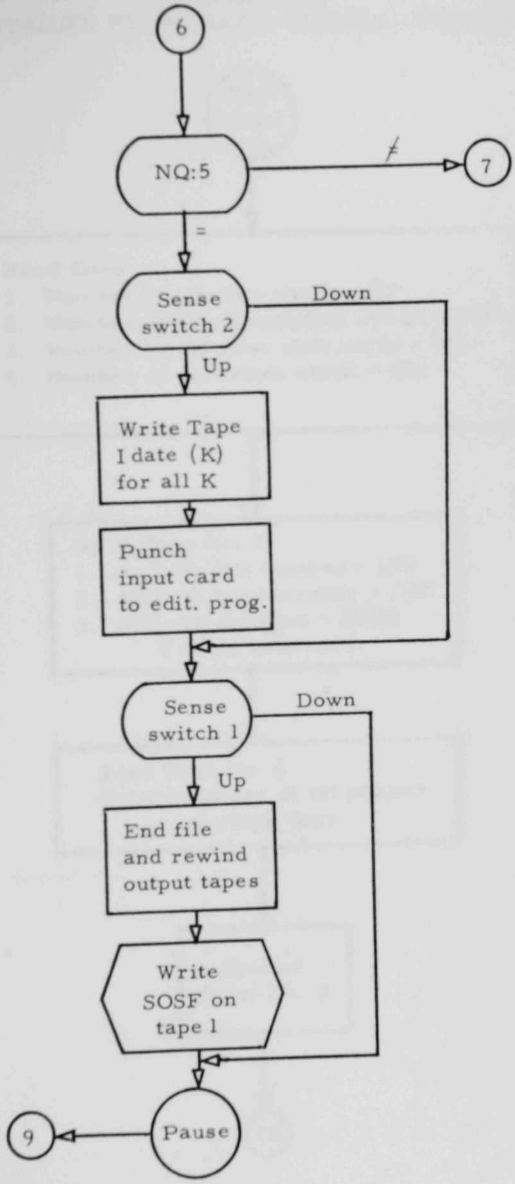
```
136     JDATEO(J,I15)=JDATE(J,K)
        MTFTO(I15)=MTFT(K)
        IF(I15-200)144, 140, 140
140     WRITEOUTPUTTAPE2, 74, (IO(L), JO(L),
       X(JDESO(M, L), M=1, 10), JDO(L),(JDATEO(M, L),
       XM=1, 4), MTFTO(L), L=1, 200)
        I15=1
        GOTO146
144     I15=I15+1
146     CONTINUE
        IF(I11-I13)152, 152, 148
148     IF((I11-I13)-200) 150, 126, 126
150     I12=I11-I13
        GOTO126
152     I15=I15-1
        REWIND4
        WRITEOUTPUTTAPE2, 74, (IO(L), JO(L),
       X(JDESO(M, L), M=1, 10), JDO(L), (JDATEO(M, L),
       XM=1,4), MTFTO(L), L=1, I15)
154     ENDFILE2
        REWIND2
        REWIND5
        STOP77777
        END(0, 1, 0, 0, 1)
```

APPENDIX E
OVERALL FLOWCHART, CRITICAL PATH PROGRAM




1
5
7
9
Read
NQ (control digit), INEW
I(K), J(K), new value of
JD(K), arbitrary comple-
tion time LJLFT(K)
NQ1
1
2
3
4
5
6
NQ
NQ6
Update
calendar
date file
NQ2
2
Compute ear-
liest starting
times, JEST
(K) for all jobs
Compute maxi-
mum value of
JEST(K). Set
all JLFT(K) =
MAX JEST (K)
NQ3,4
Compute latest
finishing times,
JLFT(K), for
all jobs.
Enter block and
make backward
pass from pres-
ent value of K
4

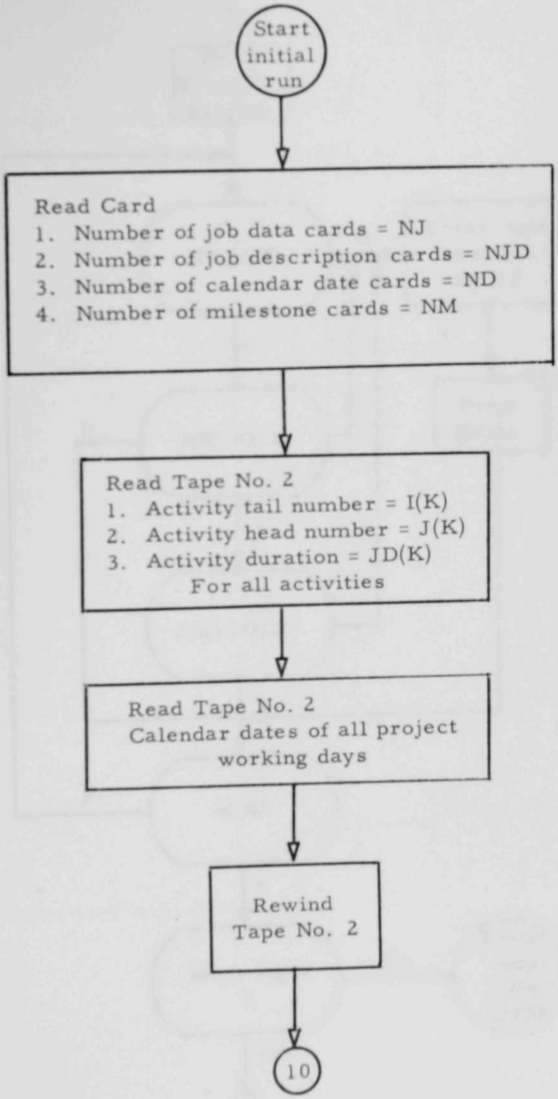

4
NQ:3
=
5
≠
Write Tape
I(K), J(K) = job description, JD(K), earliest & latest starting and finishing times & total slack. Prepare input tape for manpower computation & data editing program
Sense switch 4
Down
Up
Write Tape (For significant project milestones)
Event number, event description, earliest and latest event times, and event slack.
6



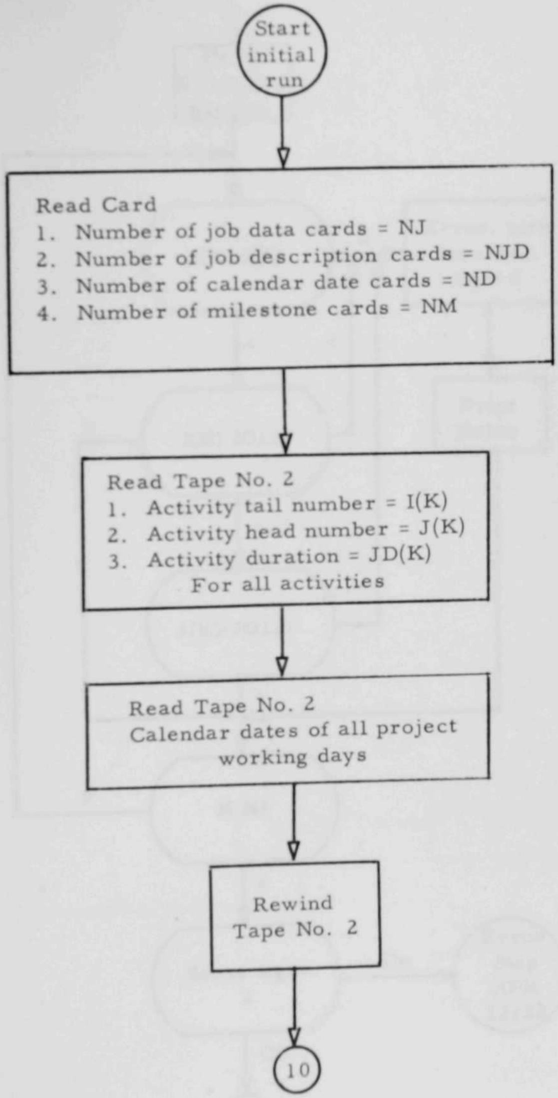
6
NQ:5
≠
7
=
Sense switch 2
Down
Up
Write Tape I date (K) for all K
Punch input card to edit. prog.
Sense switch 1
Down
Up
End file and rewind output tapes
Write SOSF on tape 1
Pause
9

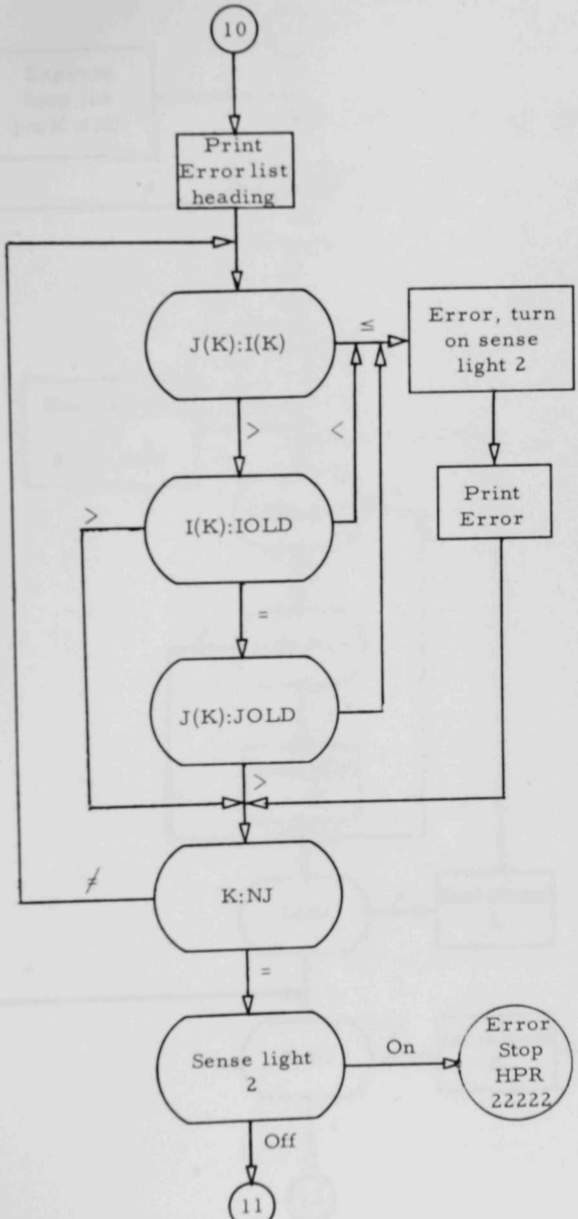
APPENDIX F

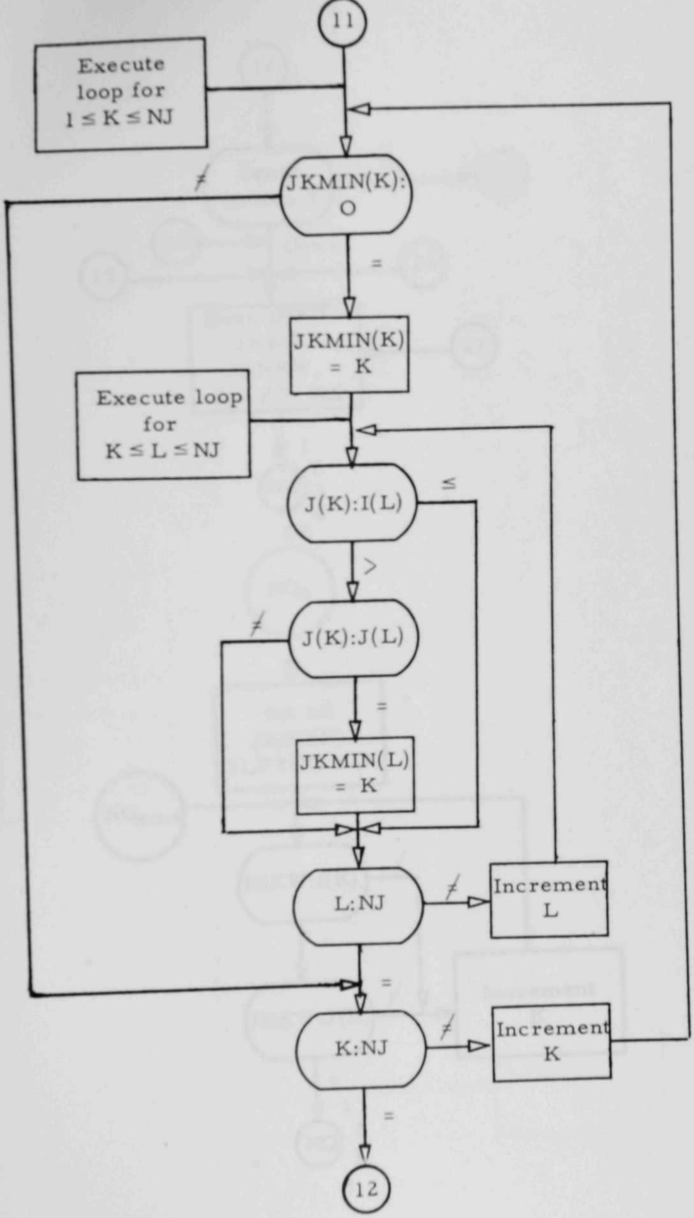
SEMI-DETAILED FLOWCHART, CRITICAL PATH PROGRAM



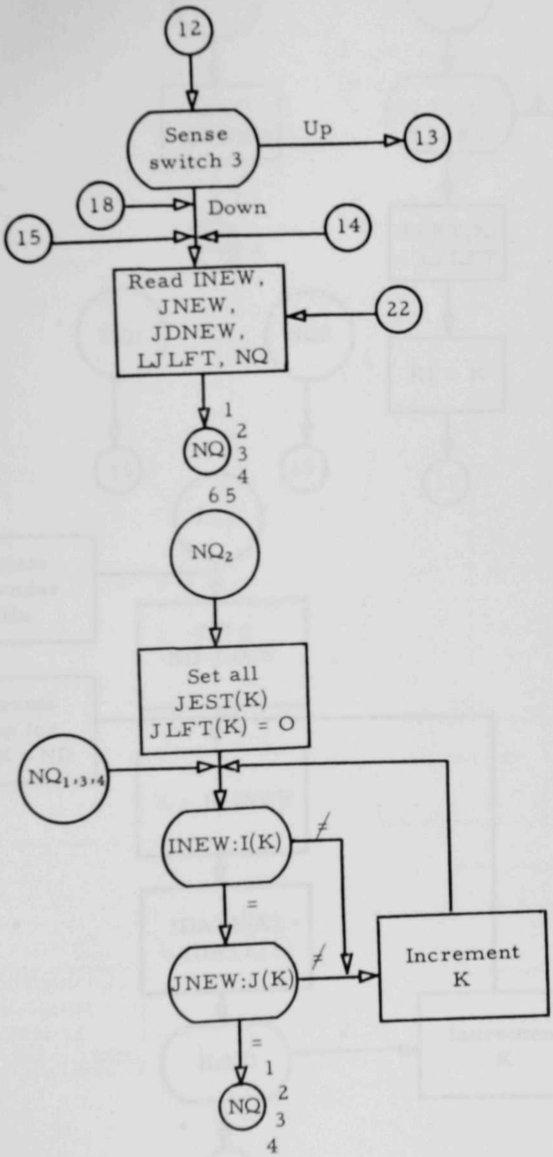
APPENDIX F
SEMI-DETAILED FLOWCHART, CRITICAL PATH PROGRAM

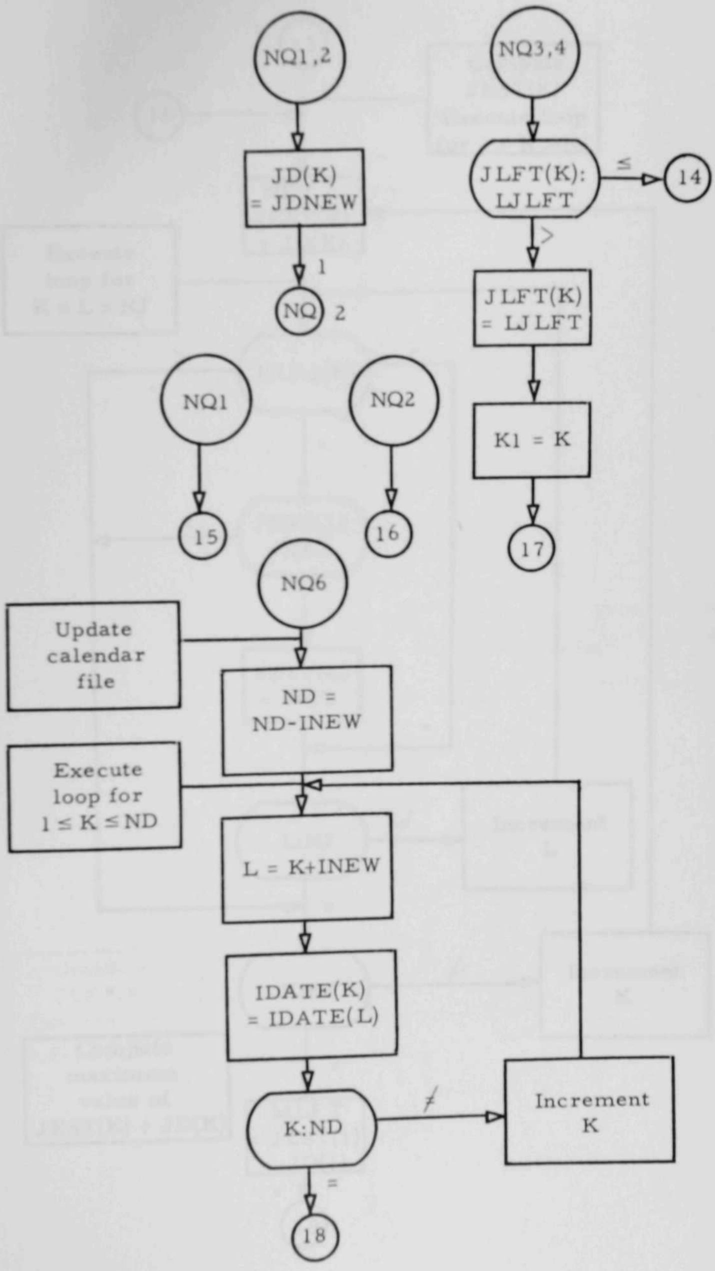


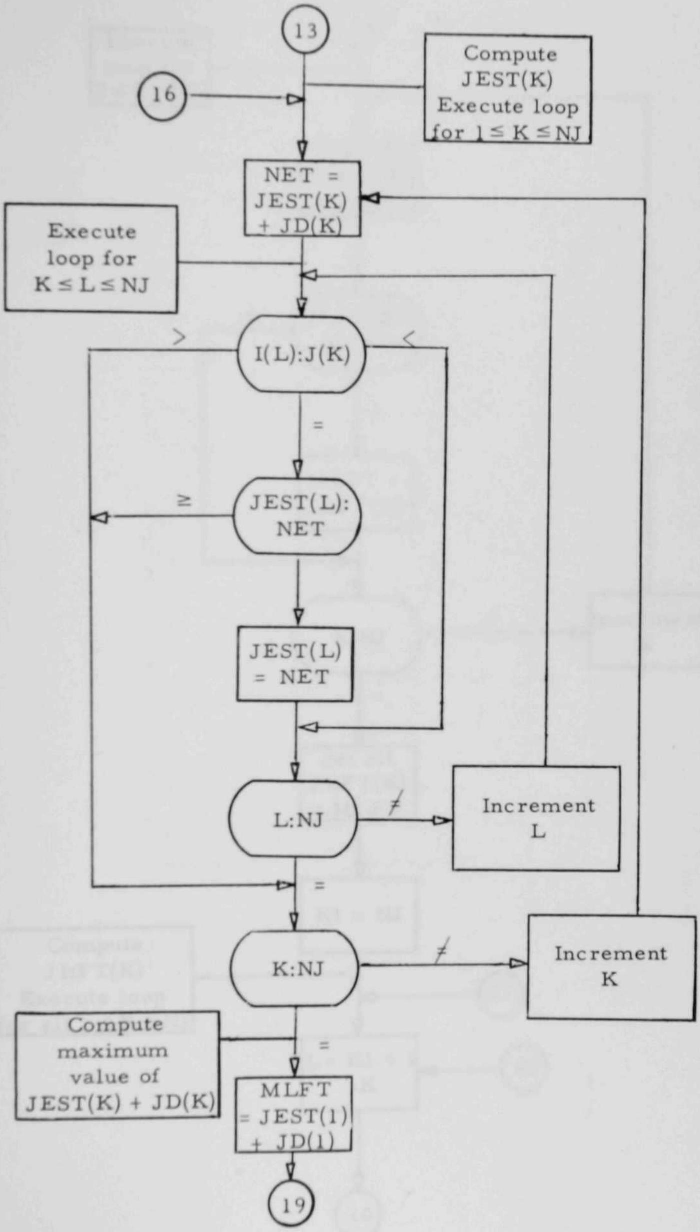

10
Print Error list heading
J(K):I(K)
≤
Error, turn on sense light 2
>
<
Print Error
>
I(K):IOLD
=
J(K):JOLD
>
≠
K:NJ
=
Sense light 2
On
Error Stop HPR 22222
Off
11



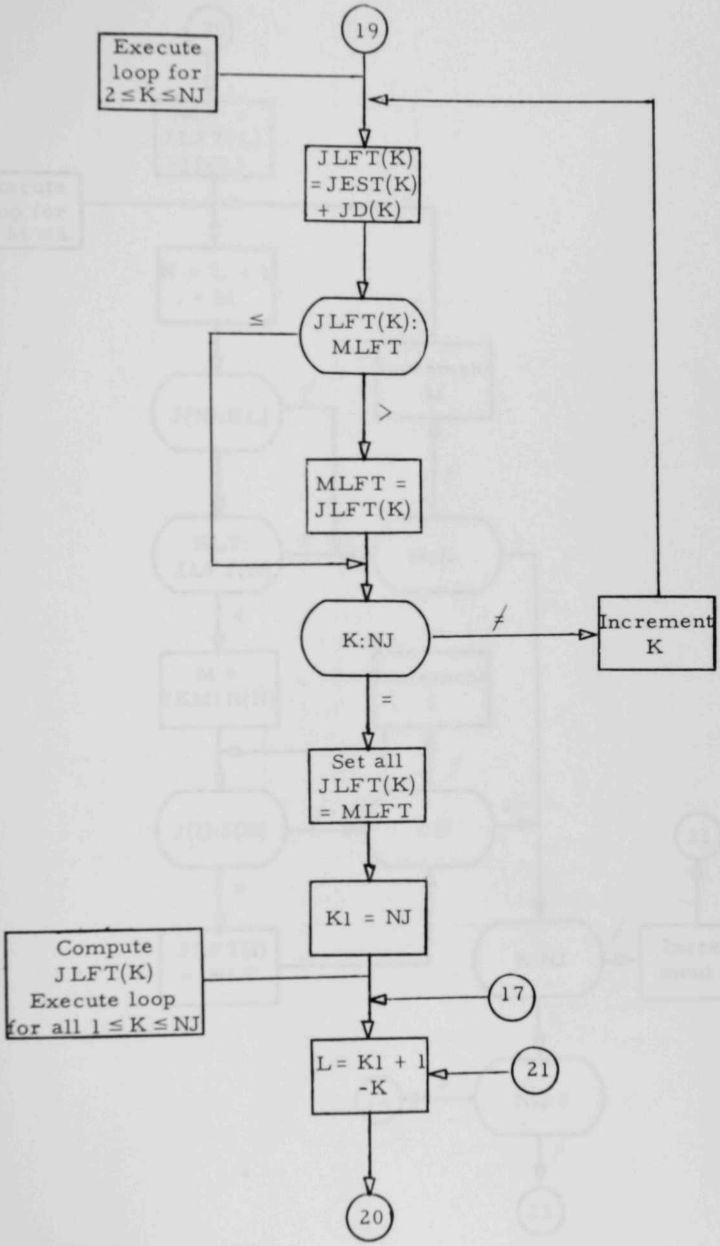
11
Execute loop for 1 ≤ K ≤ NJ
JKMIN(K): O
≠
=
JKMIN(K) = K
Execute loop for K ≤ L ≤ NJ
J(K):I(L)
≤
>
J(K):J(L)
≠
=
JKMIN(L) = K
L:NJ
≠
Increment L
=
K:NJ
≠
Increment K
=
12

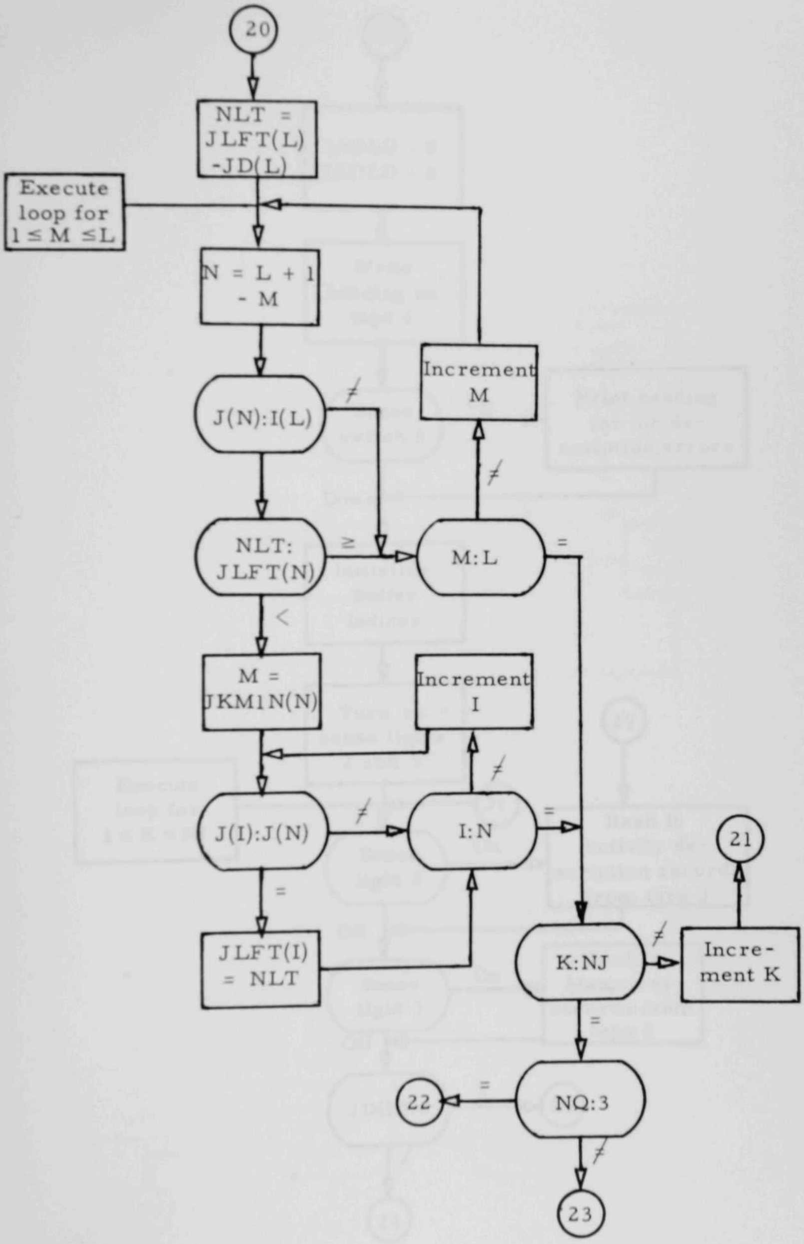

12
Sense switch 3
Up
13
18
Down
14
15
Read INEW, JNEW, JDNEW, LJLFT, NQ
22
1
2
NQ
3
4
6 5
NQ2
Set all JEST(K) JLFT(K) = O
NQ1,3,4
INEW:I(K)
≠
=
JNEW:J(K)
≠
Increment K
=
1
2
NQ
3
4

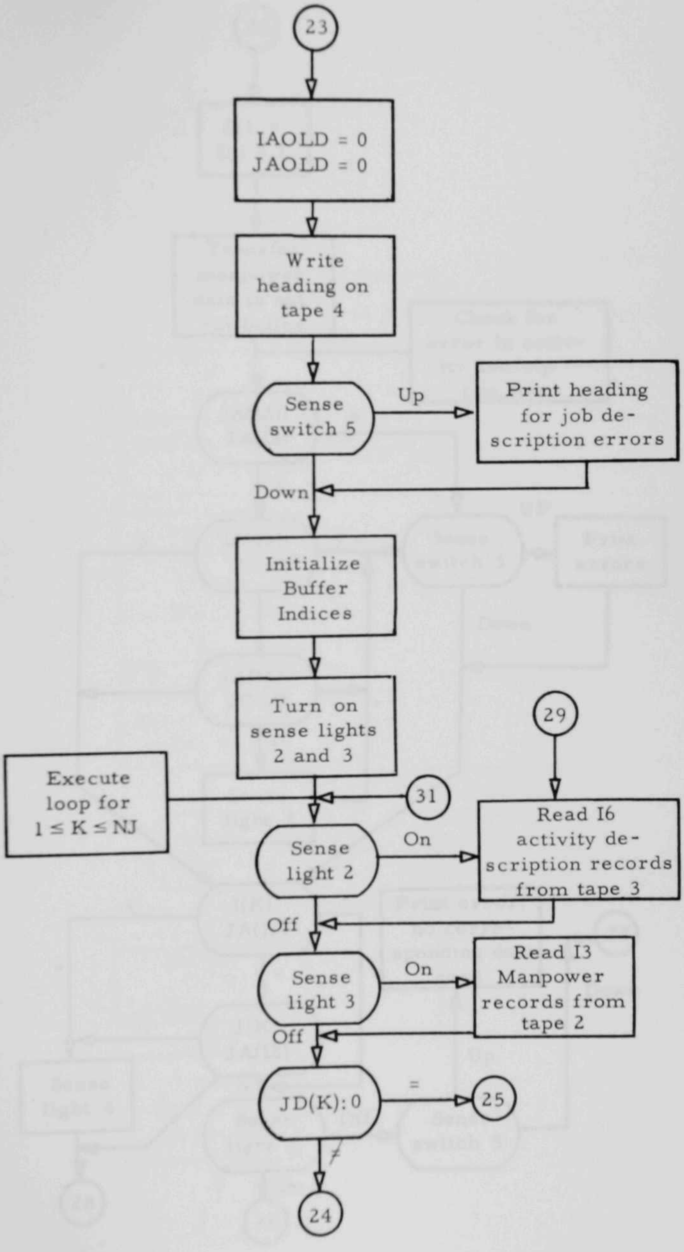

NQ1,2
JD(K)
= JDNEW
1
NQ
2
NQ3,4
JLFT(K):
LJLFT
≤
14
>
JLFT(K)
= LJLFT
K1 = K
17
NQ1
15
NQ2
16
NQ6
Update
calendar
file
ND =
ND-INEW
Execute
loop for
1 ≤ K ≤ ND
L = K+INEW
IDATE(K)
= IDATE(L)
K:ND
≠
Increment
K
=
18

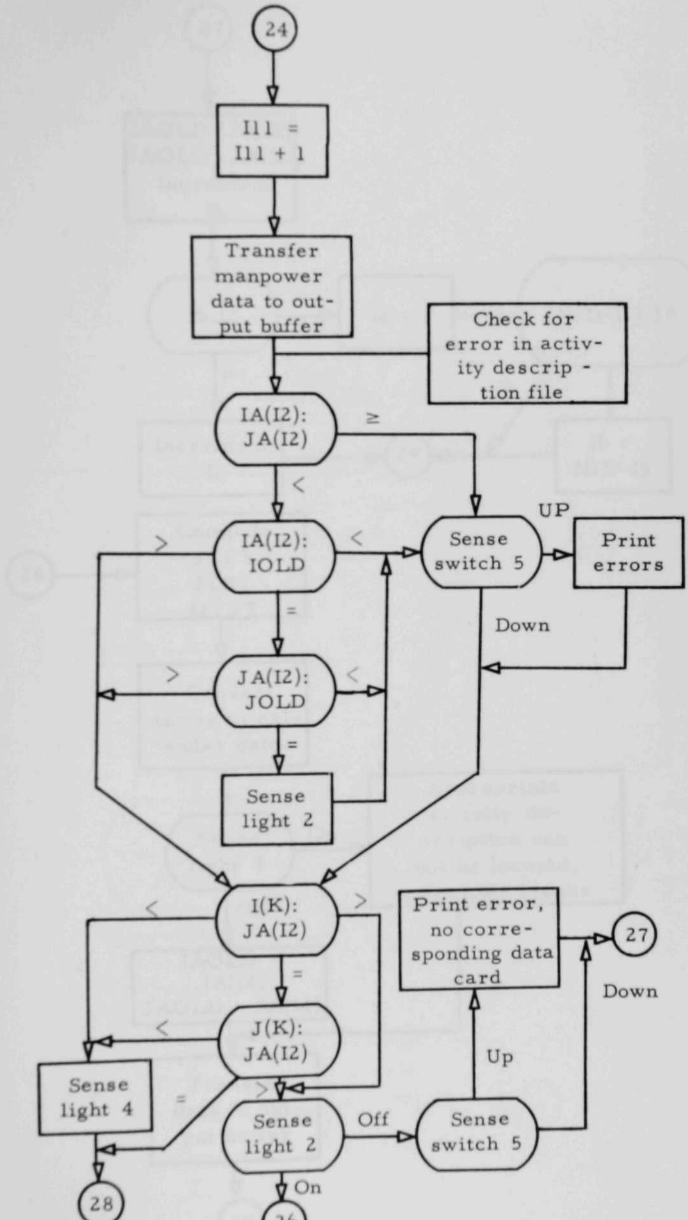


13
16
Compute JEST(K) Execute loop for 1 ≤ K ≤ NJ
NET = JEST(K) + JD(K)
Execute loop for K ≤ L ≤ NJ
>
I(L):J(K)
<
=
≥
JEST(L): NET
JEST(L) = NET
L:NJ
≠
Increment L
=
K:NJ
≠
Increment K
Compute maximum value of JEST(K) + JD(K)
=
MLFT = JEST(1) + JD(1)
19

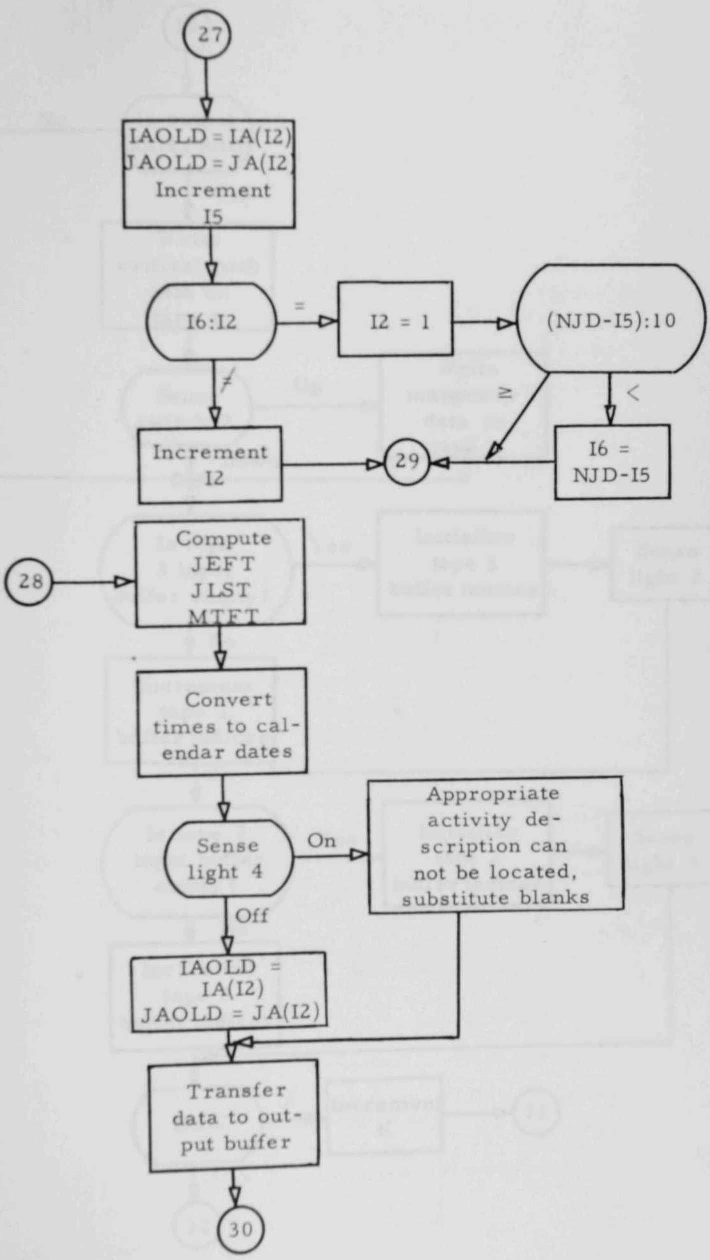

19
Execute loop for 2 ≤ K ≤ NJ
JLFT(K) = JEST(K) + JD(K)
JLFT(K): MLFT
≤
>
MLFT = JLFT(K)
K:NJ
≠
Increment K
=
Set all JLFT(K) = MLFT
K1 = NJ
Compute JLFT(K) Execute loop for all 1 ≤ K ≤ NJ
17
L = K1 + 1 -K
21
20

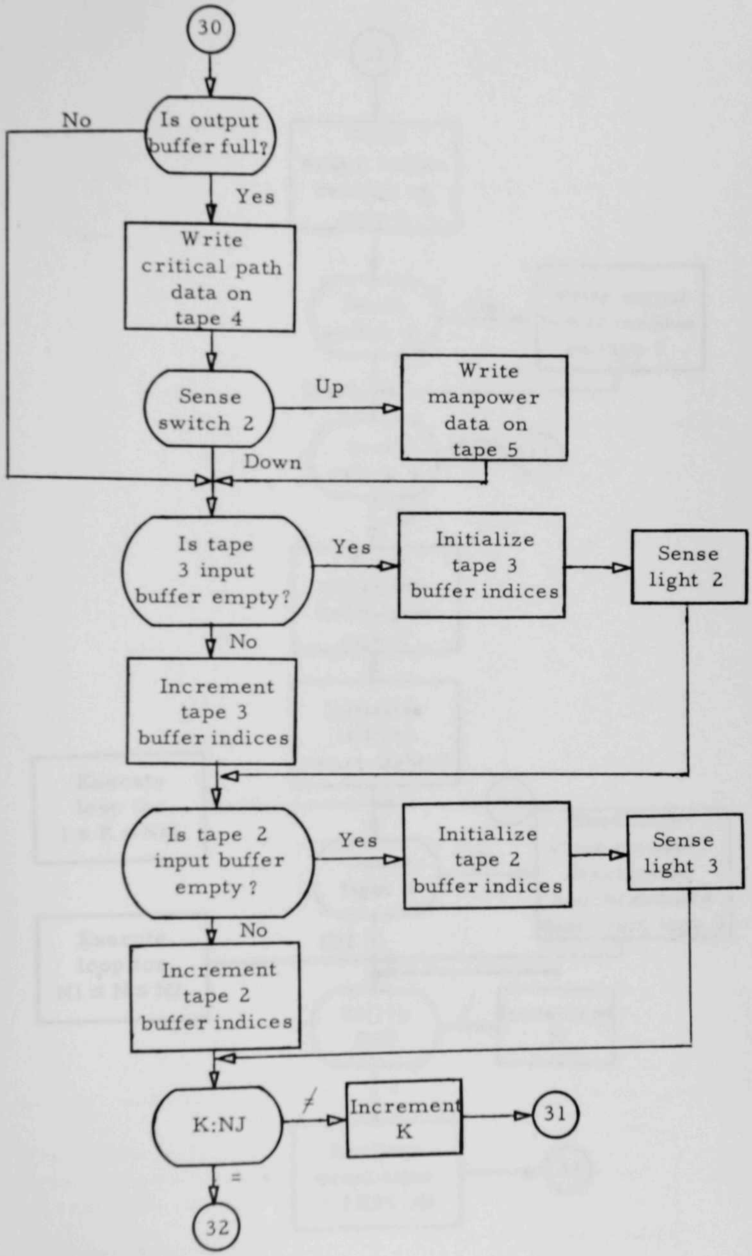

20
NLT = JLFT(L) -JD(L)
Execute loop for 1 ≤ M ≤L
N = L + 1 - M
J(N):I(L)
≠
Increment M
≠
NLT: JLFT(N)
≥
M:L
=
<
M = JKM1N(N)
Increment I
≠
J(I):J(N)
≠
I:N
=
=
JLFT(I) = NLT
21
K:NJ
≠
Incre- ment K
=
22
=
NQ:3
≠
23

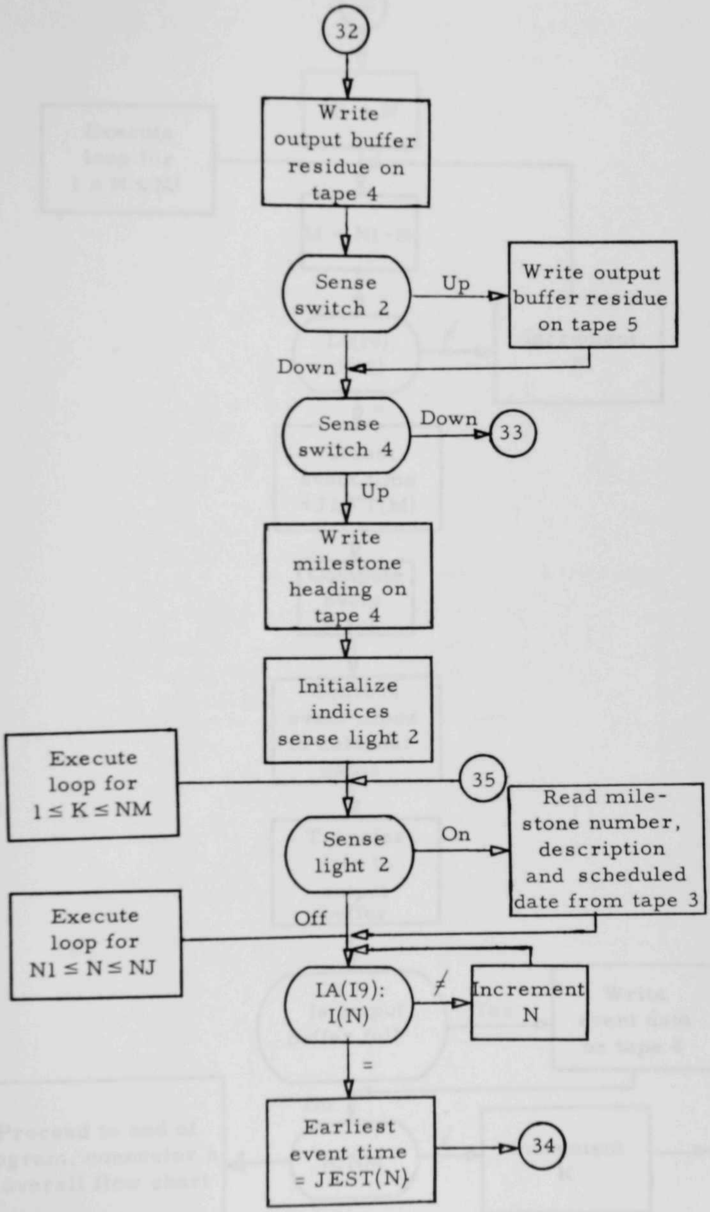

23
IAOLD = 0
JAOLD = 0
Write heading on tape 4
Sense switch 5
Up
Print heading for job description errors
Down
Initialize Buffer Indices
Turn on sense lights 2 and 3
29
Execute loop for 1 ≤ K ≤ NJ
31
Sense light 2
On
Read I6 activity description records from tape 3
Off
Sense light 3
On
Read I3 Manpower records from tape 2
Off
JD(K): 0
=
25
≠
24


24
I11 = I11 + 1
Transfer manpower data to output buffer
Check for error in activity description file
IA(I2): JA(I2)
≥
<
IA(I2): IOLD
<
>
=
Sense switch 5
UP
Print errors
Down
JA(I2): JOLD
>
<
=
Sense light 2
I(K): JA(I2)
<
>
=
J(K): JA(I2)
<
>
=
Sense light 4
Sense light 2
Off
On
Sense switch 5
Up
Print error, no corresponding data card
Down
27
28
26

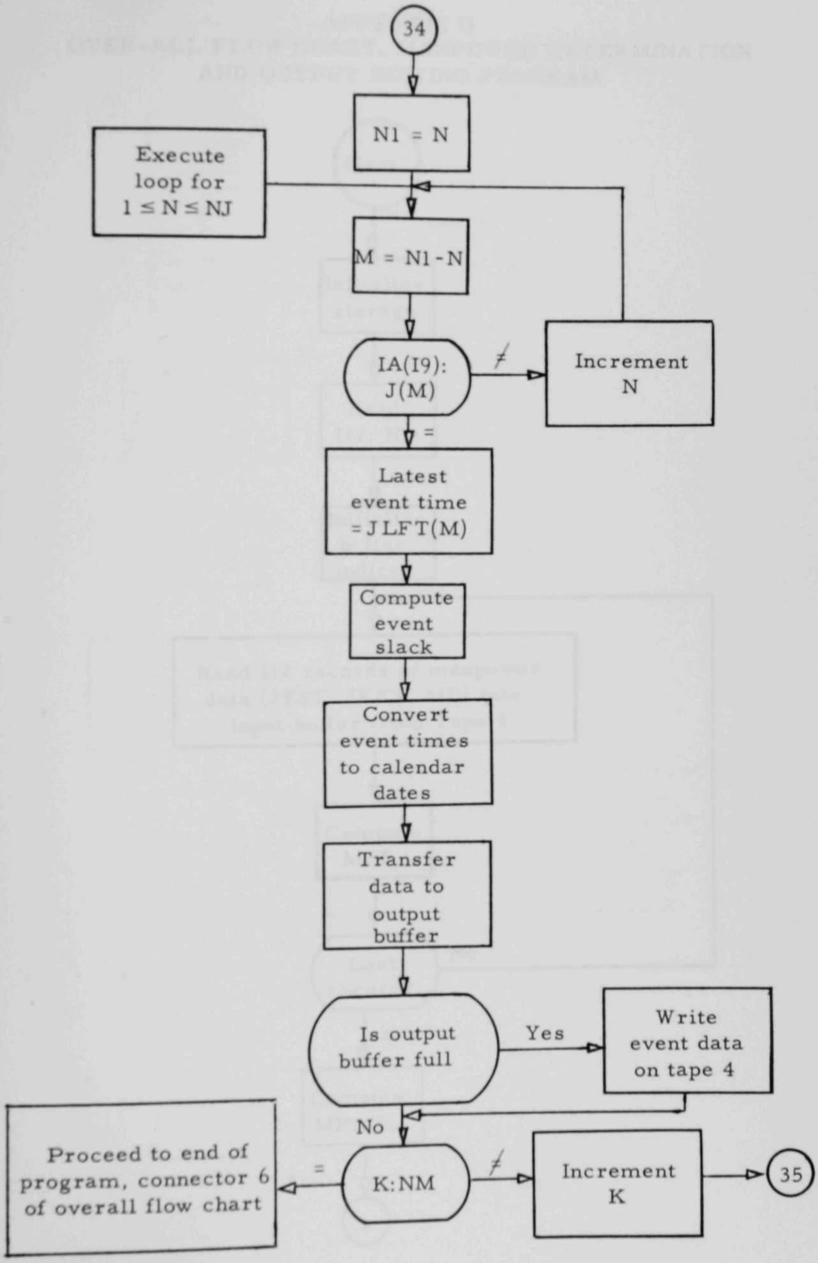
24
Transfer manpower data to output buffer
Check for error in activity description file
Print errors
Up
Down
Sense light 4
Sense light 2
Sense switch 5
Off
On
26
28
27


27
IAOLD = IA(I2)
JAOLD = JA(I2)
Increment I5
I6:I2
=
I2 = 1
(NJD-I5):10
≠
≥
<
Increment I2
29
I6 = NJD-I5
28
Compute JEFT JLST MTFT
Convert times to calendar dates
Sense light 4
On
Appropriate activity description can not be located, substitute blanks
Off
IAOLD = IA(I2)
JAOLD = JA(I2)
Transfer data to output buffer
30


30
Is output buffer full?
No
Yes
Write critical path data on tape 4
Sense switch 2
Up
Write manpower data on tape 5
Down
Is tape 3 input buffer empty?
Yes
Initialize tape 3 buffer indices
Sense light 2
No
Increment tape 3 buffer indices
Is tape 2 input buffer empty?
Yes
Initialize tape 2 buffer indices
Sense light 3
No
Increment tape 2 buffer indices
K:NJ
≠
Increment K
31
=
32

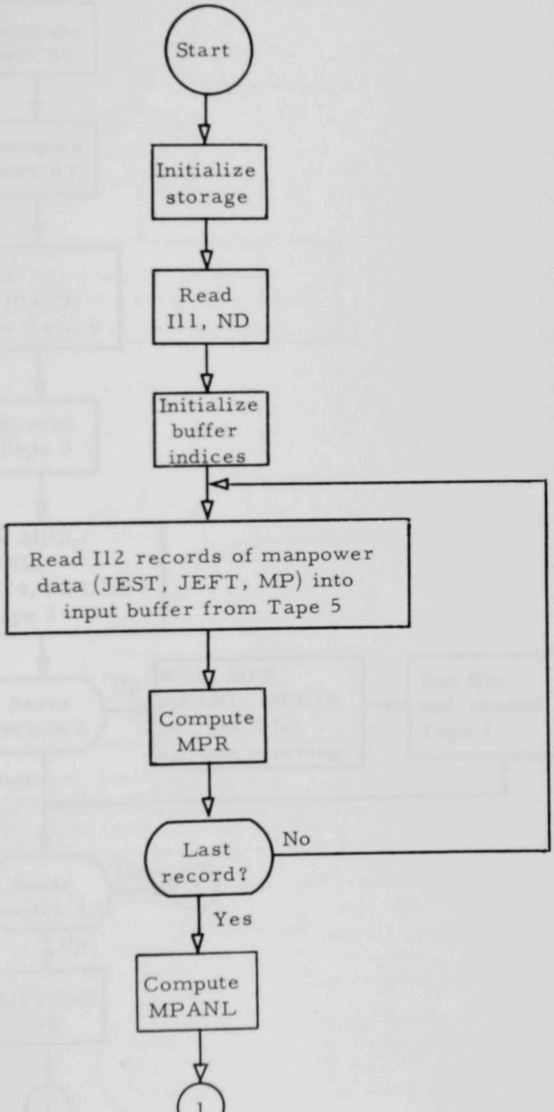


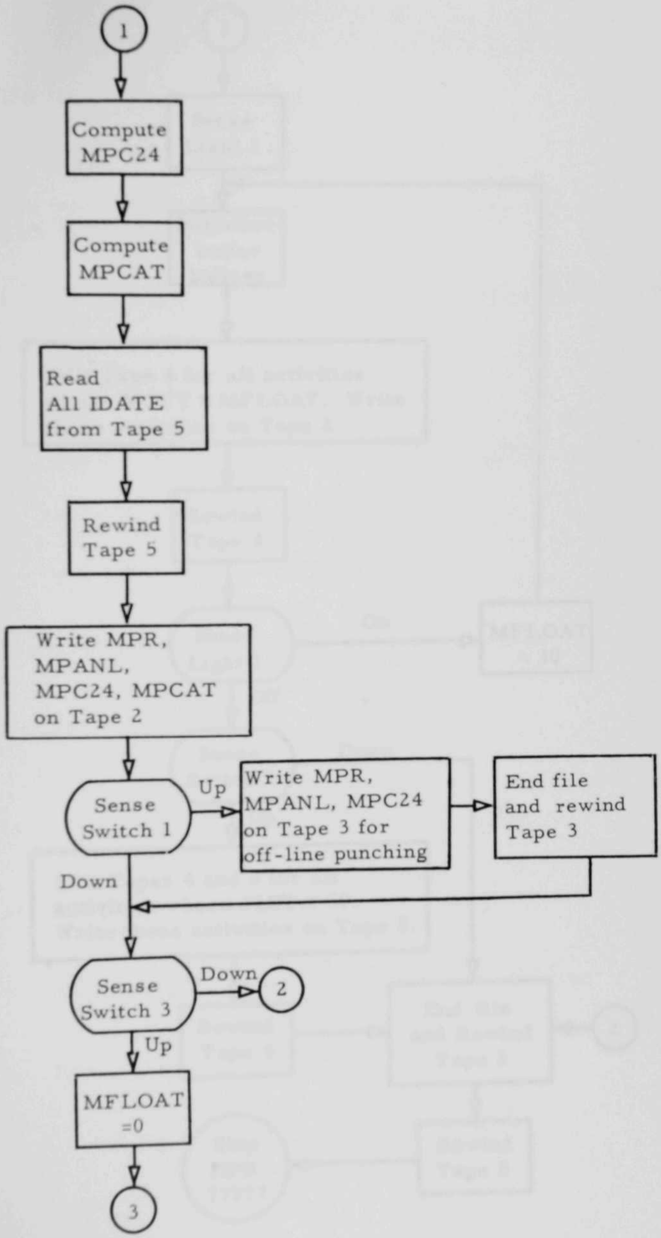
32
Write output buffer residue on tape 4
Sense switch 2
Up
Write output buffer residue on tape 5
Down
Sense switch 4
Down
33
Up
Write milestone heading on tape 4
Initialize indices sense light 2
Execute loop for 1 ≤ K ≤ NM
35
Sense light 2
On
Read milestone number, description and scheduled date from tape 3
Off
Execute loop for N1 ≤ N ≤ NJ
IA(I9): I(N)
≠
Increment N
=
Earliest event time = JEST(N)
34

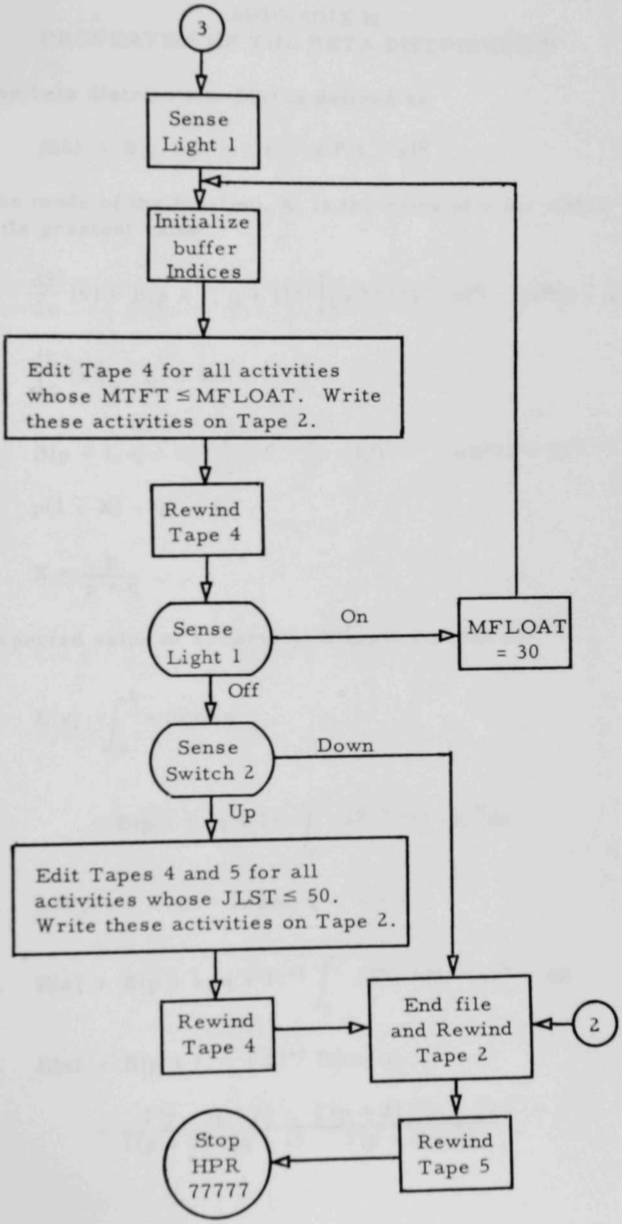


34
N1 = N
Execute loop for 1 ≤ N ≤ NJ
M = N1 - N
IA(I9): J(M)
≠
Increment N
=
Latest event time = JLFT(M)
Compute event slack
Convert event times to calendar dates
Transfer data to output buffer
Is output buffer full
Yes
Write event data on tape 4
No
Proceed to end of program, connector 6 of overall flow chart
=
K:NM
≠
Increment K
35

APPENDIX G
OVER-ALL FLOW CHART, MANPOWER DETERMINATION AND OUTPUT EDITING PROGRAM




1
Compute MPC24
Compute MPCAT
Read All IDATE from Tape 5
Rewind Tape 5
Write MPR, MPANL, MPC24, MPCAT on Tape 2
Sense Switch 1
Up
Write MPR, MPANL, MPC24 on Tape 3 for off-line punching
End file and rewind Tape 3
Down
Sense Switch 3
Down
2
Up
MFLOAT =0
3


3
Sense Light 1
Initialize buffer Indices
Edit Tape 4 for all activities whose MTFT ≤ MFLOAT. Write these activities on Tape 2.
Rewind Tape 4
Sense Light 1
On
MFLOAT = 30
Off
Sense Switch 2
Down
Up
Edit Tapes 4 and 5 for all activities whose JLST ≤ 50. Write these activities on Tape 2.
Rewind Tape 4
End file and Rewind Tape 2
2
Rewind Tape 5
Stop HPR 77777

APPENDIX H
PROPERTIES OF THE BETA DISTRIBUTION

The beta distribution $\beta(x)$ is defined as

$$\beta(x) = B(p + 1, q + 1)^{-1} x^p(1 - x)^q \quad .$$

A. The mode of the function, X, is the value of x for which $\beta(x)$ achieves its greatest value:

1. $\frac{d\beta}{dx}(x) = B(p + 1, q + 1)^{-1}\left[px^{p-1}(1 - x)^q - qx^p(1 - x)^{q-1}\right]$

2. $\frac{d\beta}{dx}(x)\Big|_{x = X} = 0$

3. $B(p + 1, q + 1)^{-1}\left[pX^{p-1}(1 - X)^{p-1} - qX^p(1 - X)^{q-1}\right] = 0$

4. $p(1 - X) - qX = 0$

5. $X = \frac{p}{p + q}$

B. Expected value of x, namely, E(x), is defined as

1. $E(x) = \int_0^1 x\,\beta(x)dx$

2. $\quad = B(p + 1, q + 1)^{-1}\int_0^1 x^{p+1}(1 - x)^q dx$

3. Let $p + 1 = m - 1$, and $q = n - 1$

4. $E(x) = B(p + 1, q + 1)^{-1}\int_0^1 x^{m-1}(1 - x)^{n-1} dx$

5. $E(x) = B(p + 1, q + 1)^{-1} B(m, n)$

$$= \frac{\Gamma(p + q + 2)}{\Gamma(p + 1)\,\Gamma(q + 1)} \frac{\Gamma(p + 2)\,\Gamma(q + 1)}{\Gamma(p + q + 3)}$$

6. We know that for the gamma function

$$\Gamma(n + 1) = n\,\Gamma(n)$$

7. Therefore $E(x) = \dfrac{\Gamma(p + q + 2)}{\Gamma(p + 1)\,\Gamma(q + 1)} \, \dfrac{(p + 1)\,\Gamma(p + 1)\,\Gamma(q + 1)}{(p + q + 2)\,\Gamma(p + q + 2)}$

8. $E(x) = \dfrac{p + 1}{p + q + 2} = \mu$

C. The variance of x, namely, VAR(x), is defined as the expected value of the second moment taken about the mean.

1. $\mathrm{VAR}(x) = E(x - \mu)^2 = E(x^2 - 2\mu x + \mu^2)$

$$= E(x^2) - 2\mu E(x) + E(\mu^2) = E(x^2) - 2\mu^2 + \mu^2$$

$$= E(x^2) - \mu^2 = \left[\int_0^1 x^2 B(p+1,\, q+1)^{-1} x^p (1 - x)^q dx\right] - \mu^2$$

2. $\mathrm{VAR}(x) = B(p + 1,\, q + 1)^{-1} \displaystyle\int_0^1 x^{p+2}(1 - x)^q dx - \mu^2$

3. Let $p + 2 = m - 1$, $q = n - 1$

4. $\mathrm{VAR}(x) = B(p + 1,\, q + 1)^{-1} \displaystyle\int_0^1 x^{m-1}(1 - x)^{n-1} dx - \mu^2$

5. $= B(p + 1,\, q + 1)^{-1}\, B(m, n) - \mu^2$

$$= \frac{\Gamma(p + q + 2)}{\Gamma(p + 1)\,\Gamma(q + 1)} \, \frac{\Gamma(p + 3)\,\Gamma(q + 1)}{\Gamma(p + q + 4)} - \mu^2$$

6. $= \dfrac{\Gamma(p + q + 2)\,(p + 2)\,(p + 1)\,\Gamma(p + 1)}{(p + q + 3)\,(p + q + 2)\,\Gamma(p + q + 2)} - \mu^2$

$$= \frac{(p + 2)\,(p + 1)}{(p + q + 3)\,(p + q + 2)} - \frac{(p + 1)^2}{(p + q + 2)^2}$$

$$= \frac{(p + 1)\,(p^2 + 4p + pq + 2q + 4 - p^2 - 4p - pq - q - 3)}{(p + q + 2)^2\,(p + q + 3)}$$

7. $\mathrm{VAR}(x) = \dfrac{(p + 1)\,(q + 1)}{(p + q + 2)^2\,(p + q + 3)}$

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